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Toward an Understanding of the Breakdown of Heat Transfer Modeling in Reciprocating Flows

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TOWARD AN UNDERSTANDING OF THE BREAKDOWN OF
HEAT TRANSFER MODELING IN RECIPROCATING
FLOWS

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Abstract

Reynolds average Navier-Stokes (RANS) modeling has established itself as a critical design tool in many engineering applications, thanks to its superior computational efficiency. The drawbacks of RANS models are well known, but not necessarily well understood: poor prediction of transition, non-equilibrium flows, mixing and heat transfer, to name the ones relevant to our study. In the present study, we use a direct numerical simulation (DNS) of a reciprocating channel flow driven by an oscillating pressure gradient to test several low- and high-Reynolds’ RANS models. Temperature is introduced as a passive scalar to study heat transfer modeling. Low-Reynolds’ models manage to capture the overall physics of wall shear and heat flux well, yet with some phase discrepancies, whereas high-Reynolds’ models fail. We have derived an integral method for wall shear and wall heat flux analysis, which reveals the contributing terms for both metrics. This method shows that the qualitative agreement appears more serendipitous than driven by the ability of the models to capture the correct physics. The integral method is shown to be more insightful in the benchmarking of RANS models than the typical comparisons of statistical quantities. This method enables the identification of the sources of discrepancies in energy budget equations. For instance, in the wall heat flux, one model is shown to have an out of phase dynamic behavior when compared to the benchmark results, demonstrating a significant issue in the physics predicted by this model. Our study demonstrates that the integral method applied to RANS modeling yields information not previously available that should guide the derivation of physically more accurate models.
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Chapter 1

Introduction

1.1 Motivation

Currently the design of internal combustion engines (ICEs) rely on computational fluid dynamic (CFD) studies alongside physical engine prototypes in order to produce efficient and powerful engines. The design and use of experimental engines is costly and due to the engine geometry accurate flow measurements are limited to regions where a probe or viewing window can be installed. On the other hand the internal combustion engine is a very challenging system to simulate from a fluid dynamics view, the flow is made up of swirling, expanding, and compressing flows, which occur cyclically within the cylinder. Along with the complex flow structures, models must be used for combustion, soot formation, and wall heat transfer due to various time and length scales.

Recently the International Energy Agency (IEA) has set out guidelines for the 2°C scenario (2DS), which looks to limit global average temperature rise by reducing fuel use per kilometer by 30-50% for all cars by 2050 (19). This requirement adds to the already high tech requirements for engine design. In order to meet the goals of the IEA, it is crucial that ICE designs and simulations can analyze various design parameters across multiple iterations in a timely manner. Unfortunately at this time, full resolution and timely simulation cannot
be accomplished together. To lower computational cost, the simulations of ICEs in industry avoid solving many of the physical processes involved by using models (29). The problem in reducing physical phenomena defined by complex, nonlinear deterministic equations to simpler models is the need for many assumptions and, in the present case, the use of ad-hoc parameterization. The cumulative effects of these approximations lead to considerable uncertainties in the simulation results. A specific example of such assumption may be found in turbulence modeling. In ICE CFD, turbulence modeling aims at obtaining a solution with a certain amount of averaging in time and space. The former removes the need to simulate many cycles and the latter relaxes the requirement that the computational grid should capture the smallest scales of the flow. Current turbulence models are derived from our statistical knowledge of equilibrium flows, i.e. flows with statistically steady boundary conditions. Since they are the only models available, all ICE CFD simulations use such models. The equilibrium hypothesis is obviously questionable, however no study has properly assessed its impact on the solution. Accordingly, this research sets out to properly assess the use of these models in simulating heat and mass transfer in highly non-equilibrium flows, comparable to those found in ICEs. We seek to determine what errors are created and the degree to which they would affect the design of an ICE.

Turbulence in general is difficult to simulate accurately, since its description is not fully defined. Turbulence is characterized by its ability to mix fluids and dissipate kinetic energy. It is comprised of a large range of scales from large energy carrying eddies, which in turn spawn smaller eddies, which spawn even smaller eddies, until the Kolmogorov limiting scale is reached, at which point the kinetic energy of the small scales is dissipated through viscosity into heat. This cascade of energy is seen in figure 1.1.
The most profound fact is that this complex process of energy transfer and dissipation is described by the Navier-Stokes' (NS) equation. For the untrained eye, turbulent motion appears to be totally random and chaotic, however turbulence is the consequence of many coherent interacting structures. For the study of ICEs, wall bounded flows are of specific interest and the turbulent structures involved are important for the efficient operation of said ICEs. Again relating to the operation of ICEs, the flow field during each cycle goes through phases of low-intensity, almost laminar, and high-intensity turbulent regimes due to the moving piston boundary. Understanding the development of transition to turbulence and its relation to the mean flow has been the topic of numerous research for many years (3)(30)(37)(39). Research on these transitional flows has determined that there exist coherent structures within the near wall region which maintain the turbulence, both producing and dissipating energy. There are various explanations for the coherent structures of near wall turbulence, however it is most commonly associated with streamwise veloc-
ity streaks and quasi streamwise vortices. Figure 1.2 shows grey isosurfaces representing the quasi streamwise vortices and the fluctuating wall shear contours on the bottom plane representing the streamwise velocity streaks from a turbulent channel flow DNS.

![Figure 1.2: Turbulent channel flow moving from lower left to upper right. Quasi streamwise vortices (grey isosurfaces) only shown for lower half of channel and streamwise velocity streaks (bottom plane contours)](image)

The process of which these streaks and vortices form has been debated for many years now, but Jimenez et al (20) determined that the streamwise vortices extract energy from the mean flow and produce alternating streaks of streamwise velocity, which in turn produce the quasi-streamwise vortices and this mechanism is the strongest within the near wall flow. They also determined that these near-wall structures obey a self-sustaining regeneration cycle, which is independent of the core flow. Relating to ICEs, these coherent structures
are responsible for momentum, mass, and thermal transfer near the walls, all of which are important factors in the prediction of the performance of an engine.

One would like to be able to fully resolve these flow characteristics for ICE design, however, it is impossible to find an analytical solution to the NS equation for a piston engine. The only way to determine the exact time and spatial flow field is to perform a direct numerical simulation (DNS), where the entire flow domain is discretized onto a mesh composed of cells, whose size is at least small enough to sufficiently capture the dissipation scale otherwise known as Kolmogorov scale. The DNS of a piston engine requires to the need for 50-100 million cells. For instance, in 2014 Schmitt et. al. (32) performed a direct numerical simulation of flow within an engine like geometry, which used spectral element method and 57.8 million cell points and 1.3 million CPU hours to simulate 8 engine cycles. This is one of the first full direct numerical simulations to fully resolve the fluid scales in an engine geometry. It is noted that this simulation is a cold flow and does not include combustion, soot formation or any other interesting phenomena. Being that this is the first time a simulation of this scale has been performed and that it only simulated the cold flow, speaks to the massive computational expense of these simulations. Currently this level of simulation is out of the question for industry needs, since it is necessary to test various design iterations all within a reasonable time for the engine design cycle. To obey this time requirement there are two other options, a large eddy simulation (LES) and a Reynolds’ Averaged Navier-Stokes’ (RANS) simulation.

The LES relies on the fact that the large scale motions of the flow are largely geometry dependent, where as the smaller eddies are more universal in nature. As such, the Navier-Stokes’ equation is filtered to only solve for the larger scales of size $\Delta$ and the smaller scales are modeled implicitly with a subgrid-scale model. With this, the cell size is now of the order $\Delta$ and doesn’t require the resolution of smaller scales. However, as was mentioned previously, the correct prediction of near wall turbulence is imperative to an accurate engine
simulation, so the accuracy is directly related to the models applied. Recently LES has been used increasingly in industry due to increases in computational power and its ability to predict CCV, (2)(23) (26). Still the large time requirements for statistical convergence paired with simulations of around 2 million cells from Liu et. al. (23) still present challenges for the use in the iterative design process, so the most common method for design and simulation is to use a RANS simulation. Unlike the DNS and LES methods, the RANS simulation solves a modified Navier-Stokes’ equation, which has been averaged in space, producing spatial average and fluctuating components. Commonly in industry, these RANS simulations are performed on coarse meshes, in order to have fast turnaround times for the design process, usually in the range 36,000-500,000 cells, (28)(36). A direct consequence of the averaging process is the reliance on semi-empirical turbulence models that carry large uncertainties. Along with turbulence modeling, the coarse meshes require near wall models, to bridge the gap from the wall to the core flow.

Reliance on these wall functions and turbulence models is of the main interest of this research, since many RANS models are known to poorly predict unsteady and non-equilibrium flows (17)(18). Surprisingly the assessment of RANS models’ performance in reciprocating flows with heat transfer has only recently been investigated (13). Common turbulence models for RANS simulations require the use of test cases to tune the model constants, which are usually equilibrium flows e.g. steady boundary layer or channel flows. The main scope of this research relates to these equilibrium based turbulence models and the errors that occur when they are applied to the simulation of an ICE. In order to perform a fundamental analysis of the models and how they behave in non-equilibrium conditions, we have reduced the complexity of the piston engine geometry while retaining the reciprocating aspect of the flow, by a simulating a reciprocating channel flow. This decision is based on the ease of computation and that given the simplicity, if the models breakdown in this flow scenario, then they cannot possibly perform better in the full engine geometry.
Our research focuses on the use of DNS reciprocating channel flows at two different flow periods as test cases. OpenFOAM is used to produce complimentary RANS simulations of a 2D reciprocating channel flow. Research of pulsating or reciprocating flows has been given close attention in recent years for its engineering and medical applications. This research has included laminar flows with heat transfer (25), (35), DNS of turbulent channels (10),(33), experimental setups ,(15),(38), as well as comparison with RANS modeling (34). Our research is motivated by the fact that little research touches upon the errors involved in the RANS modeling of non-equilibrium flows and how they relate directly to near wall predictions. Although there has been previous work on non-equilibrium models in the past (1)(31), none have shown drastic improvements over the common models. Along with an emphasis on both core flow predictions and near wall predictions, our research includes a fundamental comparison of turbulent and laminar reciprocating channel flows. In regards to the near wall predictions, we have developed integral relations to determine the contributing terms for the wall shear stress in the laminar, DNS, and RANS cases. In the same regard the contributing terms for the wall heat flux were also determined for the DNS and RANS cases. Both of these relations offer an deeper understanding to how laminar and turbulent flows differ, and clear indications where the RANS modeling is failing.
Chapter 2

Numerical Methods

2.1 Direct Numerical Simulation (DNS)

Direct numerical simulations (DNS) were made available by Dr. Yves Dubief in order to produce reference data for the RANS turbulence modeling analysis, which exactly solve the continuity and Navier-Stokes’ equation.

\[
\frac{\partial u_i}{\partial x_i} = 0
\]  

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{dp}{dx_i} + \nu \frac{\partial^2 u_i}{\partial x_j} 
\]  

The DNS is full 3D, with periodicity in both the spanwise \((y)\) and streamwise \((x)\) directions and the flow is driven by a cosinusoidal pressure gradient. The computational mesh was of size \(L_x = 10\), \(L_y = 5\), and \(L_z = 2\), with \(N_x = 128\), \(N_y = 128\), \(N_x = 129\). The cells in \(x\) and \(y\) were uniformly distributed while the cells in the wall normal direction are stretched using a tanh function and a stretch factor of 2.8. The minimum \(\Delta z\) is 0.0003 and the max \(\Delta z\) is 0.0437.

The code is based off of the in-house code NGA of Desjardins et al (9) who have de-
developed a massively parallel high order conservative finite difference scheme, which can solve flows on non-uniform Cartesian and cylindrical meshes. The time integration was performed with a second-order semi-implicit Crank-Nicolson. The spatial integration uses a conservative second order finite volume method. Since the simulation of a reciprocating flow involves varying flow scales, the time step was variable from $0.0005 \leq \Delta t \leq 0.02$ and the data was saved every $T/32$ for 10 simulation periods producing 320 data files. Given that the channel is periodic in the spanwise and streamwise directions, the data can be average in x and y, and it is also phase averaged over the 10 simulation periods allowing for converged statistics.

2.2 Reynolds’ Averaged Navier-Stokes (RANS) Simulation

In order to increase computation speeds at a cost of detail, the Reynolds’ averaged Navier-Stokes (RANS) turbulence approach is used, which applies Reynolds’ decomposition to the full Navier-Stokes equation, where the exact flow solution through time and space is averaged to produce an average and fluctuating component.

\[ u_i(x_i, t) = \overline{u_i}(x_i) + u_i'(x_i, t) \tag{2.3} \]

where,

\[ \overline{u_i}(x_i) = \lim_{T \to \infty} \frac{1}{T} \int_0^T u_i(x_i, t) dt \tag{2.4} \]

Here $T$ is an averaging interval that is large compared to the time scale of the turbulent fluctuations. This averaging is only applicable to steady state flows and if an unsteady flow is to be solved an ensemble average must be used instead. To produce an ensemble average,
the flow is averaged over identical micro states within the flow. Within the following research individual phases of the oscillating period are used as the ensembles for averaging.

\[ N_\phi = \sum_{i=1}^{N} \delta (i \Delta t, \phi) \]  

(2.5)

and

\[ \bar{u}_i (x_i, \phi) = \frac{1}{N_\phi} \sum_{i=1}^{N_\phi} u_i \delta (i \Delta t, \phi) \]  

(2.6)

where \( \delta \) is the Kronecker delta and \( N_\phi \) is the number of phases. To derive the RANS equation we apply the Reynolds’ averaging process to first the continuity equation,

\[ \frac{\partial \bar{u}_i}{\partial x_j} = 0. \]  

(2.7)

Continuing with the process, the incompressible momentum equation is averaged, however this is more difficult due to the non-linear convective term, the mean of the left hand side of the momentum equation is,

\[ \frac{D \bar{u}_i}{Dt} = \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i u_j)}{\partial x_j}. \]  

(2.8)

Applying the Reynolds’ average decomposition to the non-linear term produces,

\[ \bar{u}_i u_j = (\bar{u}_i + u'_i)(\bar{u}_j + u'_j) \]

\[ = \bar{u}_i \bar{u}_j + u'_i \bar{u}_j + u'_i \bar{u}_j + \bar{u}_i u'_j \]

\[ = \bar{u}_i \bar{u}_j + u'_i \bar{u}_j + u'_i \bar{u}_j + u'_i u'_j \]

\[ = \bar{u}_i \bar{u}_j + u'_i u'_j, \]

since

\[ \bar{u}'_j \bar{u}_i = \bar{u}_i u'_j = 0. \]  

(2.9)

(2.10)
Combining equation 2.8 and 2.9 and since the flow is considered incompressible we arrive at,

\[
\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + \bar{u}_j \frac{\partial u_i}{\partial x_j} + \frac{\partial (u'_i u'_j)}{\partial x_j}
\] (2.11)

performing the average on the rest of the terms of the momentum equation is easy since the spatial derivative commutes with the average operation, doing so, results in the Reynolds’ average Navier-Stokes (RANS) equation.

\[
\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial (u'_i u'_j)}{\partial x_j}
\] (2.12)

Equation 2.12 can be rewritten slightly as,

\[
\rho \left( \frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} \right) = \frac{\partial}{\partial x_j} \left[ -\bar{p} \delta_{ij} + \mu \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) - \rho (u'_i u'_j) \right]
\] (2.13)

The terms within the square bracket represent the stresses in the flow, mean pressure field stress, viscous stress, and fluctuating velocity or Reynolds’ stresses. The Reynolds’ stress is a symmetric second-order tensor, with the normal stresses down the diagonal and the shear stresses on the off diagonal. It is possible to define a transport equation for the Reynolds’ stress, however as is seen in equation 2.14.

\[
\frac{\partial u'_i u'_j}{\partial t} + \bar{u}_k \frac{\partial u'_i u'_j}{\partial x_k} = -\frac{\partial}{\partial x_k} \left[ u'_i u'_j u'_k + \frac{\bar{p}}{\rho} \left( \delta_{kj} u'_i + \delta_{ki} u'_j \right) \right] + \frac{\partial}{\partial x_k} \left[ \nu \frac{\partial (u'_i u'_j)}{\partial x_j} \right] - \left( u'_i u'_k \frac{\partial \bar{u}_j}{\partial x_k} + u'_j u'_k \frac{\partial \bar{u}_i}{\partial x_k} \right) + \frac{\bar{p}}{\rho} \left( \frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i} \right) - 2\nu \frac{\partial u'_i}{\partial x_k} \frac{\partial u'_j}{\partial x_k}
\] (2.14)
However this equation involves highly complex double and triple correlations, which are far too complex for current computing power. In order to close the Reynolds’ stress term it must be modeled. The most common modeling practice is to use the a turbulent viscosity, which was introduced by Boussinesq in 1877 and is as follows,

\[-u'_i u'_j = \nu_T \left( \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij}, \tag{2.15}\]

where \(\nu_T\) is the turbulent viscosity, and substituting into equation gives,

\[\frac{\partial \tau_{ij}}{\partial t} + \frac{\partial \tau_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \nu_{eff} \left( \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_i} \right) \right] - \frac{1}{\rho} \frac{\partial}{\partial x_i} \left( p + \frac{2}{3} \rho k \right), \tag{2.16}\]

where the effective viscosity \(\nu_{eff}\) is the sum of the regular and turbulent viscosity \((\nu + \nu_T)\). In order to close equation 2.16, we require the use of a turbulence model to calculate the turbulent eddy viscosity \(\nu_T\). The base method of closure for \(\nu_T\) relies on the modeling of turbulent scales and solving the associated transport equations. One of the most commonly used models is the \(k - \varepsilon\) model, which requires \(k\) the turbulent kinetic energy which is defined as half the trace of the Reynolds stress tensor and the dissipation rate of turbulent kinetic energy \(\varepsilon\). \(k\) is defined as follows,

\[k = \frac{1}{2} u'_i u'_j \tag{2.17}\]

The turbulent kinetic energy has the following transport equation.

\[\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \nu_T \left( \frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_i} \right) \frac{\partial \tau_{ij}}{\partial x_j} - \varepsilon. \tag{2.18}\]

Here we see \(k\) and \(\varepsilon\) are coupled and we require a transport equation for \(\varepsilon\), which is derived directly from equation 2.18 and is as follows,
\[
\begin{align*}
\frac{\partial \varepsilon}{\partial t} + \mathbf{u}_j \frac{\partial \varepsilon}{\partial x_j} &= \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \varepsilon}{\partial x_j} \right) + C_{\varepsilon 1} \frac{\varepsilon}{k} \nu_T \left( \frac{\partial \mathbf{u}_i}{\partial x_j} + \frac{\partial \mathbf{u}_j}{\partial x_i} \right) \frac{\partial \mathbf{u}_i}{\partial x_j} - C_{\varepsilon 2} \frac{\varepsilon^2}{k}.
\end{align*}
\] (2.19)

Given these transport equation, the classic eddy viscosity closure can be formed,

\[
\nu_T = C_{\mu} \frac{k^2}{\varepsilon}
\] (2.20)

now equation, 2.16 is fully closed.

2.2.1 **Near Wall Turbulence Modeling**

Within the scope of this research, we investigate the use of two different kinds of turbulence models, called high Reynolds’ and low Reynolds’, referring not to the specific Reynolds’ number of the flow, but whether the near wall region is modeled or fully resolved. The high Reynolds’ models rely on coarser meshes and wall models to bridge the gap between the wall and the first cell point. These high Reynolds’ models are popular in the commercial industry for their low computational costs, but they are often times applied to types of flow where the major assumptions are not valid. The low Reynolds’ models do not rely on wall models since the meshes are fully resolved in the near wall region, but most do necessitate the use of damping functions for the eddy viscosity to maintain correct near wall turbulence behavior. The use of low Reynolds’ models requires more computational efforts, however it is of the interest for this research to determine whether the gains in accuracy outweigh the increase in computational complexity. Figure 2.1 shows a schematic of the difference between high and low Reynolds’ meshes.
The main assumption for the high Reynolds’ models is that the region between the first cell and the wall is well defined by a logarithmic velocity profile. The key to the high Reynolds’ mesh is to have the cell point sufficiently far from the wall that it is located in the fully turbulent regime, where this log law region is known to occur. Given this criteria, the wall functions apply the boundary conditions for $\overline{u}$, $k$, and $\varepsilon$, allowing for a vastly smaller cell count in the near wall region. For our simulations, we have used three different high-Reynolds models, which are the $k-\varepsilon$, $k-\omega$ and $k-\omega SST$ models. For the $k-\omega$ models, equation A.2 can be transformed into the transport equation for $\omega$, which is the specific rate of turbulent dissipation defined as,

$$\omega = \frac{\varepsilon}{kC_\mu} \quad (2.21)$$

and requires a slightly modified $k$ transport equation and its own transport equation,

$$\frac{\partial k}{\partial t} + \overline{u}_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \right) \frac{\partial k}{\partial x_j} \right] + \nu_T \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \frac{\partial \overline{u}_i}{\partial x_j} - \beta^* k \omega \quad (2.22)$$
\[ \frac{\partial \omega}{\partial t} + \pi_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu T}{\sigma_k} \right) \frac{\partial \omega}{\partial x} \right] + \frac{\omega}{k} \left[ \nu T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} \right] - \beta \omega^2 \]  

(2.23)

The corresponding turbulent viscosity \( \nu_T \) is defined as,

\[ \nu_T = \frac{k}{\omega} \]  

(2.24)

When full resolution in the near wall region is possible, equations 2.18 and A.2 are known to produce inaccurate results in the near wall region, so for most low Reynolds’ models a damping function must be applied to correct these inaccuracies. There are many variations of damping functions, however for our research we are only interested in the Launder-Sharma variation of the \( k - \varepsilon \) equation. This model is close to the original \( k - \varepsilon \) model, with some slight variation. First a new variable \( \bar{\varepsilon} \) is introduced,

\[ \varepsilon = \bar{\varepsilon} + D \quad D = 2\nu \left( \frac{\partial \sqrt{k}}{\partial n} \right)^2 \]  

(2.25)

where \( n \) is the wall normal direction. The advantage of \( \bar{\varepsilon} \) is the natural boundary condition \( \bar{\varepsilon} = 0 \) at the walls. Launder & Sharma(Need Citation) also proposed the addition of the following term to the \( \varepsilon \) equation.

\[ E = 2\nu \nu_T \left( \frac{\partial^2 \pi}{\partial y^2} \right)^2 , \]  

(2.26)

which compensates for additional production to further balance the diffusion and dissipation in the vicinity of the walls. Given the new variable and term, the \( k \) and \( \varepsilon \) transport equations can be rewritten.
\[ \frac{\partial k}{\partial t} + \pi_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \right) \frac{\partial k}{\partial x_j} \right] + \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} - (\bar{e} + D). \]  

(2.27)

\[ \frac{\partial \bar{e}}{\partial t} + \pi_j \frac{\partial \bar{e}}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\nu_T}{\sigma_e} \frac{\partial \bar{e}}{\partial x_j} \right) + C_{\varepsilon 1} \frac{\bar{e}}{k} \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} - C_{\varepsilon 2} f \frac{\bar{e}^2}{k} + E. \]  

(2.28)

The turbulent viscosity is redefined as,

\[ \nu_T = C_{\mu} \mu \frac{k^2}{\bar{e}}. \]  

(2.29)

An approximation for the \( D \) term is as follows,

\[ D = 2\nu \left( \frac{\partial \sqrt{k}}{\partial n} \right)^2 \approx 2\nu \left( \nabla \sqrt{k} \right)^2. \]  

(2.30)

There also exists a special extension to the low Reynolds \( k-\varepsilon \) models, called the \( k-\varepsilon-v^2-f \) or more commonly just \( v^2-f \), which along with \( k \) and \( \varepsilon \) equations, solves the transport of the square of the fluctuating wall normal velocity \( v^2 \), which allows for a new turbulence scale that better represents the damping of turbulence near the wall. It also solves an elliptic blending equation \( f \), which model the anisotropic wall effects. The main advantage of this model is its ability to be solved up to the wall without the need for an ad-hoc damping function. Along with the standard \( k \) and \( \varepsilon \) equations, The \( v^2 \) equations is as follows,

\[ \frac{\partial v^2}{\partial t} + \pi_j \frac{\partial v^2}{\partial x_j} = k f - \frac{v^2}{k} \varepsilon + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_T}{\sigma_{v^2}} \right) \frac{\partial v^2}{\partial x_j} \right] \]  

(2.31)

which requires the solution of the elliptic relaxation function \( f \), which is as follows,
\[ L^2 \nabla^2 f - f = \frac{C_1 - 1}{T} \left( \frac{\nu^2}{k} - \frac{2}{3} \right) - C_2 \frac{P_k}{\varepsilon} \] (2.32)

where \( L \) is the turbulence length scale,

\[ L = C_L \max \left[ \frac{k^{3/2}}{\varepsilon}, C_\eta \left( \frac{\nu^3}{\varepsilon} \right)^{1/4} \right] \] (2.33)

and the \( T \) is the turbulence time scale,

\[ T = \max \left[ \frac{k}{\varepsilon}, C_T \left( \frac{\nu}{\varepsilon} \right)^{1/2} \right] \] (2.34)

and finally with the new turbulence scale \( \overline{\nu^2} \) and time scale \( T \) the eddy viscosity is defined as,

\[ \nu_T = C_\mu \overline{\nu^2} T \] (2.35)

The full definition of the models used in this study are listed in appendix A.1.

### 2.3 OpenFOAM

The unsteady Reynolds averaged Navier-Stokes (URANS) calculations were done using OpenFOAM (Open Field Operation and Manipulation) an open source computational fluid dynamics (CFD) toolbox developed by OpenCFD Ltd. at ESI Group and distributed by the OpenFOAM Foundation. It utilizes a finite volume discretization to solve partial differential equations on structured and unstructured 3D meshes. OpenFOAM is highly flexible for use in simulations due to its top-level code, which allows each equation solved to be written in a tensorial notation, and specific discretization schemes can be chosen for each individual equation. It has a modular make up in which collections of functionality (e.g. numerical methods, meshing, physical models...) are compiled into their own shared library.
It includes a variety of solvers built in that cover anything from discrete molecular solvers to conjugate heat transfer solvers, as well as pre- and post-processing functions. For the present work the solver pimpleFoam was modified to add an oscillating pressure gradient and temperature as a transported scalar.

PimpleFoam is a transient turbulent incompressible flow solver, which normally utilizes the PIMPLE algorithm, which is a combination of the Semi-Implicit Method for Pressure Linked Equations (SIMPLE) and Pressure implicit with splitting of operator (PISO), however for the following reciprocating channel cases, the simple algorithm has been ignored, so it only uses the PISO method.

The simulations in this thesis were performed on OpenFOAM-2.3.0, which was the latest version during the commencement of this research.

2.3.1 Solver Configuration

Producing the reciprocating channel only required the addition of one term in the velocity equation of the solver as so,

The UEqn.H is edited as follows,

```plaintext
// Solve the Momentum equation
tmp<fvVectorMatrix> UEqn
{
    fvm::ddt(U)
    + fvm::div(phi, U)
    + turbulence->divDevReff(U)
    ==
    fvOptions(U)
    + APuls*cos(Freq*runTime.time().value())
};
UEqn().relax();
fvOptions.constrain(UEqn());
volScalarField rAU(1.0/UEqn().A());
if (pimple.momentumPredictor())
{
    solve(UEqn()) == -fvc::grad(p);
}
```
As is clear, the \(-dp/dx\) is replaced by \(APuls \ast \cos(\omega t)\) with \(APuls\) and \(Freq\) declared in the transportProperties file. To add temperature to the solver, the a TEqn.H file is created to be solved,

```plaintext
{
    alphat = turbulence->nut()/Prt;
    alphat.correctBoundaryConditions();

    volScalarField alphaEff("alphaEff", turbulence->nu()/Pr + nut()/Prt);

    fvScalarMatrix TEqn
    {
        fvm::ddt(T)
        + fvm::div(phi, T)
        - fvm::laplacian(alphaEff, T)
        == fvOptions(T)
    });

    TEqn.relax();
    fvOptions.constrain(TEqn);
    fvOptions.correct(T);
}
```

Given these two modifications the solver was ready to produce the correct reciprocating channel flow. Tables 2.1 and 2.2 list the specific schemes and solution control used for the simulations.
<table>
<thead>
<tr>
<th>Scheme</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>ddtSchemes</td>
<td>crankNicolson 1</td>
</tr>
<tr>
<td>gradSchemes</td>
<td>Gauss Linear</td>
</tr>
<tr>
<td>divSchemes</td>
<td>Vector: Gauss limitedLinearV 1</td>
</tr>
<tr>
<td></td>
<td>Scalar: Gauss limitedLinear 1</td>
</tr>
<tr>
<td></td>
<td>Tensor: Gauss linear</td>
</tr>
<tr>
<td>laplacianSchemes</td>
<td>Gauss linear corrected</td>
</tr>
<tr>
<td>interpolationSchemes</td>
<td>linear</td>
</tr>
<tr>
<td>snGradSchemes</td>
<td>corrected</td>
</tr>
</tbody>
</table>

*Table 2.1: fvSchemes for OpenFOAM simulations*
<table>
<thead>
<tr>
<th>Variable</th>
<th>Keyword</th>
<th>Entry</th>
</tr>
</thead>
<tbody>
<tr>
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<td>solver</td>
<td>GAMG</td>
</tr>
<tr>
<td></td>
<td>tolerance</td>
<td>1e-7</td>
</tr>
<tr>
<td></td>
<td>relTol</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>smoother</td>
<td>DICGaussSeidel</td>
</tr>
<tr>
<td></td>
<td>cacheAgglomeration</td>
<td>true</td>
</tr>
<tr>
<td></td>
<td>nCellsInCoarsestLevel</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>agglomerator</td>
<td>faceAreaPair</td>
</tr>
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<td></td>
<td>mergeLevels</td>
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<td></td>
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<td></td>
<td>relTol</td>
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<td></td>
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</tr>
<tr>
<td>(u</td>
<td>k</td>
<td>\varepsilon</td>
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<td>k</td>
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<tr>
<td></td>
<td>tolerance</td>
<td>1e-7</td>
</tr>
<tr>
<td></td>
<td>relTol</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.2: fvSolutions for OpenFOAM simulations
The specific transport properties were defined such that the simulation would be non-dimensional like the DNS simulation, table 2.3 defines the necessary properties of the flow.

<table>
<thead>
<tr>
<th>Contant</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>$1/2000$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$1$</td>
</tr>
<tr>
<td>T_ref</td>
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</tr>
<tr>
<td>Pr</td>
<td>0.7</td>
</tr>
<tr>
<td>PrT</td>
<td>0.9</td>
</tr>
<tr>
<td>Freq</td>
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<tr>
<td></td>
<td>$T_{40} = 0.15708$</td>
</tr>
<tr>
<td>APuls</td>
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</tr>
</tbody>
</table>

*Table 2.3: Transport properties for OpenFOAM simulations*

In order to accurately capture and average the phase statistics, it was necessary to define the simulation control parameters listed in table 2.4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>T30</th>
<th>T40</th>
</tr>
</thead>
<tbody>
<tr>
<td>startTime</td>
<td>High-Re</td>
<td>450</td>
<td>600</td>
</tr>
<tr>
<td>start</td>
<td>Low-Re</td>
<td></td>
<td></td>
</tr>
<tr>
<td>endTime</td>
<td>High-Re</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Co.</td>
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<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\Delta t$</td>
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<td>0.0003</td>
<td>.001</td>
</tr>
<tr>
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<td>3125</td>
<td>1250</td>
</tr>
</tbody>
</table>

*Table 2.4: controlDict for OpenFOAM simulations*

Using these values, the simulation will save the flow variables at every 32nd phase for 15 simulation periods, which will be shown in 2.3.4 to be the convergence requirement.
2.3.2 Mesh

For our RANS simulations we used three different structured grids, with clustering towards the walls for the low-Reynolds’ model mesh. Pointwise mesh generation software was used to produce these meshes, although OpenFOAM does offer native meshing. The simulations required three different meshes to comply with the common cell spacing requirements for the different turbulence models. For RANS modeling using high Reynolds’ models, it is common practice to define the first cell spacing such that $30 < y^+ < 200$, where $y^+ = yu_\tau/nu$, however due to the varying wall velocity during the reciprocation, the value of $u_\tau$ changes as a function of time, so the mesh was designed such that the grid was within the correct $y^+$ range during the phase with the largest velocity. Since two different periods were simulated, the high-Reynolds’ models required two different meshes in order to obey the spacing requirement. For the low-Reynolds’ models the near wall region is fully resolved and the common practice is to define the first cell spacing such that $y^+ \leq 1$. For the low-Reynolds’ models besides the two flow periods for consideration we also needed to take into account the ability to perform a grid independence study, which used data from (16) for a steady turbulent channel of $Re_\tau = 950$, where $Re_\tau = hu_\tau/\nu$ is the friction velocity Reynolds number. From the DNS reciprocating channel data it was determined that the maximum $Re_\tau$ value encountered for either flow period was around 700, so it was decided that a mesh would be created for a value of $Re_\tau = 1000$ so as to fulfill the $y^+$ requirements for the reciprocating and validation cases. The RANS meshes used by OpenFOAM are false 3-D meshes, it is a 2-d mesh with one cell depth. Since the flow is periodic, only 6 cells were used in the streamwise direction. The final mesh dimensions for the high-Reynolds’ models were $Nx = 6$, $Ny = 40$, $Nz = 1$ and $Nx = 6$, $Ny = 50$, $Nz = 1$ for period 30 and 40, respectively. The low-Reynolds’ mesh dimension were $Nx = 6$, $Ny = 256$, $Nz = 1$ for both periods.
2.3.3 Mesh Independence

As was previously mentioned to perform the mesh independence study for the low-Reynolds’ models the DNS data from (16) for $Re_\tau = 950$ was used since the value 950 was larger than the max $Re_\tau$ encountered in either flow period. Four different meshes were tested, using varying cell counts in the wall normal direction, each maintained the same first cell spacing to keep the value of $y^+ \leq 1$. The varying cell counts are as follows, $Ny = 64$, $Ny = 128$, $Ny = 256$, and $Ny = 384$ for meshes 1-4 respectively. Figures 2.2 and 2.3 show the normalized velocity profiles for both low-Reynolds’ models with the corresponding increase in mesh resolution results.

![Graph showing mesh independence for Launder-Sharma k-ε model, Re_τ = 950 steady turbulent channel](image)

*Figure 2.2: Mesh independence for Launder-Sharma k-ε model, Re_τ = 950 steady turbulent channel*
Figure 2.3: Mesh independence for $v^2 - f$ model, $Re_\tau = 950$ steady turbulent channel

As one can see, as the wall normal cell count is increased the velocity profiles collapse upon each other for both low-Reynolds' models showing that the solution has become independent of the mesh cell count. Since we see convergence of the solution between $Ny = 256$ and $Ny = 384$, we decided to use the mesh 3 with $Ny = 256$.

2.3.4 STATISTICAL CONVERGENCE

For the RANS simulations, it was necessary to verify that the simulations had reached a converged state. In order to track the convergence of the statistics from period to period, the wall shear stress was tracked at each phase as a function of simulation period. Figure 2.4 shows the wall shear value for phase 1 as a function of simulation period for each of the five models tested.
Figure 2.4: Statistical convergence for all models tested, wall shear stress as a function of simulation period

It was found that convergence was reached around the fifteenth simulated period. Accordingly all of the statistics for the RANS models were taken from period 15.

2.3.5 Boundary and Initial Conditions

Within OpenFOAM each variable to be solved is predefined for both initial and boundary conditions. Table 2.5 lists the applied initial and boundary conditions for all the variables for both simulated periods.
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</tr>
</tbody>
</table>

Table 2.5: Boundary and initial conditions for OpenFOAM simulations

2.4 Flow Visualization Techniques

The analysis of different flow scenarios by use of average profiles, can be good for gaining a basic understanding of the flow, but often times some important information is lost in one
dimensional profiles. For our research, relating specifically to turbulent structures we like to be able to visualize these structures by calculating two topological variables Q and R. Initially derived by Chong et al. (6), starting with the velocity gradient tensor $A_{ij} = \nabla u$, we can derive the characteristic equation for the eigenvalues

$$A_{ij} = \frac{\partial u_i}{\partial x_j}$$

(2.36)

$$\det[A - \lambda I] = 0$$

(2.37)

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0$$

(2.38)

(2.39)

where,

$$P = -tr[A]$$

(2.40)

$$Q = \frac{1}{2}(P^2 - tr[A^2])$$

(2.41)

$$R = -\det[A]$$

(2.42)

to gain further insight we can rewrite the definition of Q,

$$Q = \frac{1}{2}(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$$

(2.43)

where,

$$\Omega_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$$

and

$$S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

(2.44)

meaning that Q is a balance of the rotation rate and the strain rate of the flow. This implies that a region of positive Q represents a region dominated by rotation and negative Q represents a region dominated by strain. This quantity is useful in identifying regions
where a vortex most likely exists. Isosurfaces of positive Q can be plotted to visualize these coherent structures. These surfaces can be seen in figure 1.2 as grey isosurfaces.

Another technique using Q and R is to perform a joint probability density function of the two value over a specified wall-normal range. This produces data on a Q-R map like that seen in figure 2.5, which lends insight into the probability of different types of flow structure.

![Q-R Joint Probability Density Function](image)

*Figure 2.5: Q and R joint probability density function flow structure quadrants*

These JPDFs will be seen later in section 3.8 to determine the flow structures which produce different levels of turbulent stress.
CHAPTER 3

RECIPROCATING CHANNEL FLOW

Our analysis of the reciprocating channel flow begins with a fundamental comparison of a laminar reciprocating channel and a turbulent reciprocating channel. It has been determined that the flow in the turbulent case experiences varying levels of turbulence during the flow period, spanning from low-intensity turbulence to high-intensity turbulence. Since the flow approaches a laminar regime at times, gaining an understanding of the fundamental differences will help to accurately assess why the Reynolds Average Navier-Stokes’ (RANS) modeling of these flows may be falling short.

3.1 Wall Shear Stress

Let us begin by focusing our attention on the effect of turbulence on the wall shear stress within this reciprocating channel. The wall shear stress is defined as,

$$\tau_w = \nu \frac{\partial \sigma}{\partial z}$$  \hspace{1cm} (3.1)

Figures 3.1 and 3.2 show the phase averaged values of wall shear stress comparing the DNS and laminar cases and the difference of the two profiles respectively.
Interestingly we note that as the flow begins to accelerate from phases 1 to 6 the shear stress in the turbulent case is smaller than the laminar, however it increases at a faster rate, until phase 6, where we see the direct numerical simulations (DNS) and laminar profiles collapse momentarily. After phase 8 the flow velocity begins to decrease in magnitude, which produces a slower decrease in wall shear for the DNS. Near phase 10 we see the
minimum rate of decrease of wall shear for the DNS, which continues until phase 12 where
the shear begins to decrease faster again until phase 16, where the flow begins to reverse
direction and the same dynamics occur in the negative streamwise direction.

Visualizing the departure from laminar can also be done by comparing the wall shear
stress and the pressure gradient phase plot in figure 3.3.

Here we notice that there is a distinct phase difference between the applied pressure
gradient and the wall shear stress. What is most interesting is that for about half of the
flow period the turbulent and laminar profiles are well correlated. The addition of turbulence
is clear and produces an increase in the lag during the late acceleration and decreases the
lag during the early deceleration. Given the striking similarity of the turbulent case to the
laminar case a direct comparison of the velocity profiles will be performed to see how alike
the flows are.
3.2 Mean Properties

3.2.1 Velocity Profiles

A comparison of mean velocity profiles for the first half of the flow period comparing the DNS and corresponding laminar solution are shown in Figure 3.4. The left plot shows phases 1-8 and the right shows phases 9-16.

Before we begin the analysis, it is important to define a unique feature of oscillatory flows, the Stokes’ layer, which forms in between the core and near wall flow. Its thickness is dependent on the viscosity and oscillation frequency, defined as,

$$\delta_s = \sqrt{\frac{2\nu}{\omega}}$$

(3.2)

The profile of a laminar reciprocating flow is commonly associated with velocity profiles that exhibit a plug like flow, where the core region of the flow is largely undisturbed and nearly constant and in the near wall region, the existence of a Stokes’ layer is clear. The
Stokes’ layer gradually increases in distance away from the wall as the flow continues towards peak velocity magnitude at phase 8. As the flow slows after phase 8, the intensity of the original Stokes’ layer inflection point begins to decrease. Continuing to slow towards the point of reversal, a new near wall Stokes’ layer appears around phase 13, which continues to grow as the flow reverses and the process repeats identically into the reversed flow of the second half of the flow period. Comparing these profiles to the DNS, we see the how the influence of the turbulence effects the velocity profiles. With the presence of near wall turbulence, the apparent size of the plug-like region is decreased drastically, as the turbulent eddies transfer momentum away from the near wall and into the core flow. This transfer is noted by the apparent smoothing out of the Stokes’ layer peaks. We note a very close agreement with the core region velocities at period 30. Looking back to the wall shear stress plots, it is incredible that with the lack of the Stokes’ layer, the DNS near wall profiles maintain the same slope as the laminar for a majority of the flow period. Now that we have covered the directly comparable statistics, we will continue to look into the features of the DNS reciprocating channel.

Remembering that our research relates to the modeling of non-equilibrium flows in industry. An Important feature of the DNS results to look at is whether any of the profiles contain an existence of the common logarithmic law profile. The industry standard CFD modeling relies on the use of wall models to simulate a variety of flow types. These wall models are derived from from the assumption of a logarithmic law for the velocity. In combustion engine applications where reciprocating flows are encountered, the industry still relies on the use of these wall models and as such it begs the question whether these highly non-equilibrium flows exhibit the standard log law. The classical log-law plot is seen in figure 3.5 along with an log-law indicator function defined as,

\[
\text{IndFn} = z^+ \frac{dU^+}{dz^+}
\]  

(3.3)
which will show a plateau in its profile when the original function reaches a defined slope, which for our case is listed as $1/\kappa$.

![Figure 3.5: Velocity profiles as a function of $z^+$, Period 30](image1)

![Figure 3.6: Indicator function as a function of distance from wall. Period 30](image2)

Figures 3.5 and 3.6 show various phases from the first half period of the flow. The thick
dashed line in figure 3.5 represents the common log-law in equation 3.4 defined below and the dashed line in 3.6 represents the slope of that equation,

\[ U^+ = \frac{1}{0.41} \ln(z^+) + 5.25 \]  

(3.4)

It is clear from these profiles that the assumption of a logarithmic law for a reciprocating flow is not acceptable since very few of the profiles even approach the standard log-law. It must be noted that for phases 4-12 in figure 3.5 there may exist a log-law with a different slope and intercept that the data collapses to, however it is vastly different than the standard values. To expand on this log-law comparison, we will plot the same profiles for a slightly longer period of \( T = 40 \) in figures 3.7 and 3.8.

Figure 3.7: Velocity profiles as a function of \( z^+ \). Period 40
3.3 Flow Visualizations

Here we see that with the longer period we begin to see a few phases that slightly resemble the standard log-law, however the majority of phases are still far from aligning with the law. This result is expected because as the period of the flow increases, we see an increased production of turbulence and a longer duration of the turbulent period. It is understood that as the period increases towards infinity the flow progresses towards a steady state turbulent channel. Unfortunately, for internal combustion engine simulations our period of 30 is actually slower than the commonly encountered periods of an engine so it is fair to assume even less phases of the flow will approach the log-law behavior.
regions where vortices are most likely located. Along with the Q-criterion, the bottom plane of the following figures show the fluctuating wall shear stress, giving good identification of streamwise velocity streaks. These features are seen in Figures 3.9 and 3.10

**Figure 3.9:** Reciprocating channel visualization for $Q=2$ at phase $\phi = 8$. Flow moving from lower left to upper right. Bottom plane shows fluctuating wall shear stress. Side planes show streamwise velocity. Period 30
Figure 3.10: Reciprocating channel visualization for $Q=2$ at phase $\phi = 15$. Flow moving from lower left to upper right. Bottom plane shows fluctuating wall shear stress. Side planes show streamwise velocity. Period 30

These visualizations lend deep insight into what is physically occurring during the different stages of the flow. At phase 8, visualized in figure 3.9 the flow has reached its maximum forward velocity and as is denoted by the grey isosurfaces, the vortex structures are large in size and extend in the streamwise direction. From the bottom plane we see large elongated streaks exist as well. Comparing this with phase 15 in figure 3.10 when the flow is at the end of the deceleration phase, we see much smaller vortex structures have formed, and there are considerably more of them. Although it is tough to see, the presence of long streamwise velocity streaks have disappeared and only small spotty regions of fluctuating wall shear remain. The isosurfaces in both figures are at the same value of $Q = 2$, so clearly there is
more turbulent activity at phase 15 than phase 8. This is in agreement with what is seen in
the wall shear stress figure 3.1. Along with these visualizations we can plot the invariants
of velocity anisotropic tensor.

3.4 Turbulence Intensities

The turbulence intensities for the first and second half of the period are plotted in figures
3.11 - 3.13

![Figure 3.11: Streamwise turbulence intensity as a function of $z/h$. Figure A. $\phi = 1 - 8$ and figure
B. $\phi = 9 - 16$. Period 30](image)

As the flow begins its forward movement at phase 1, the streamwise turbulence intensity
is small with a slight increase from the region $0.1 < y/h < 0.4$. The intensity in this region
begins to decrease as the flow moves forward and a peak of turbulence intensity begins to
form around $y/h \approx 0.05$, due to increasing levels of shear in the stokes layer. This peak
turbulence continues to increase following the growth of the stokes layer towards phase 8 at the maximum forward velocity. After the max is reached and the flow begins to slow down, the turbulence intensity continues to increase past phase 8 reaching a maximum value around 0.7 at phase 12. Comparing the shape of the profiles before and after the maximum, we notice when the velocity is slowing down the inner core begins to retreat, while the stokes’ layer region continues moving forward, this produces the sharper tip visible for phases after the retreat. By the end of the first half of the period, we see the streamwise turbulence intensity return to the same state found in the beginning and the profiles for the second half are equal to those seen for the first half.

![Graph](image)

*Figure 3.12: Spanwise turbulence intensity as a function of z/h. Figure A. φ = 1 – 8 and figure B. φ = 9 – 16. Period 30*
Figure 3.13: Wall-Normal turbulence intensity as a function of \( z/h \). Figure A. \( \phi = 1 - 8 \) and figure B. \( \phi = 9 - 16 \). Period 30

The spanwise and wall normal turbulence intensities behave similarly to each other, but different from the streamwise turbulence intensity. As the flow begins to speed up we see a decrease in the overall spanwise and wall normal turbulence intensities. During the acceleration the streamwise turbulence intensity increases as streaks begin to form within the stokes layer, and accordingly the turbulence becomes dominated by streamwise turbulence and the spanwise and wall-normal values decrease. This decrease continues until the flow reaches its maximum forward velocity, where the spanwise and wall-normal turbulence begins to increase. It is during this time that the streamwise streaks begins to slow down and become unstable. It is known that within reciprocating flows, the streaks eventually become largely unstable and break apart. The bursting of the streamwise streaks is what produces the increasing values of spanwise and wall-normal turbulence, however at the timing of the peak values of spanwise and wall-normal fluctuations, the streamwise
turbulence in the stokes' layer is still large, but lacking in the majority of the flow. By phase 16 the streamwise turbulence is at a minimum, while the spanwise and wall-normal turbulence are still relatively large. As the flow reverses after phase 16, the process repeats itself again.

3.5 Reynolds’ Shear Stress

One of the main features unique to this reciprocating flow is the behavior of the Reynolds’ shear stress \((-\overline{u'w'})\). Figure 3.14 shows the value of Reynolds’ shear for the first half of the flow period.

![Graph A](image1.png)

![Graph B](image2.png)

Figure 3.14: Reynolds’ shear stress as a function of \(z/h\). Figure A. \(\phi = 1–8\) and figure B. \(\phi = 9–16\). Period 30

As the velocity begins to increase at phase 1 we see that there is a small positive peak of Reynolds stress around \(y/h \approx 0.05\) followed by a large region of negative values all
the way towards the channel half height. This peak value continues to increase as the
flow progresses towards phase 8, following a similar path as the peak value of streamwise
turbulence intensity seen previously. Interestingly moving towards the channel half height,
the majority of the Reynolds’ stress are still negative. This is an important fact, which
will be covered later in regards the evolution of the wall shear stress. After phase 8 when
the flow begins to slow down, the Reynolds stress continues to increase reaching a peak of
roughly 0.04 at $y/h \approx 0.1$ at phase 13. The varying states of turbulence intensities found
within this flow set the stage for differing states of turbulence, which will be analyze in the
following section.

### 3.6 Flow Anisotropy

For a deeper look into the states of turbulence involved in this flow we will reference the
work of Lumley (24) and plot the anisotropy map with the second and third invariants of
the velocity anisotropy tensor for each phase of the flow.

\[ b_{ij} = \frac{\langle u'_i u'_j \rangle}{\langle u'_k u'_k \rangle} \]  

(3.5)

which has three invariants.

\[
\begin{align*}
I & = 0 \\
II & = -\frac{b_{ij} b_{ij}}{2} \\
III & = \frac{b_{ij} b_{jk} b_{ki}}{3} - \frac{\delta_{ij}}{3}
\end{align*}
\]

(3.6)

To produce the anisotropy map, we plot $-II$ vs. $III$, with three bounding lines, which
are as follow, on the right hand side,

\[
III = 2\left(\frac{II}{3}\right)^{3/2}
\]

(3.7)

on the left hand side its opposite
\[ III = -2\left(\frac{I}{3}\right)^{3/2} \quad (3.8) \]

and on the top,

\[ III = \frac{-9I}{27} - 1 \quad (3.9) \]

It is known that all turbulent states must fall within this bounding region. The left corner represents an axisymmetric 2-D turbulent region where one component of the turbulent has a smaller magnitude than the other two, which is described as pancake-like. The right corner represents the other kind of axisymmetric 2-D turbulence where one component is larger than the other two, described as cigar-like. The top corner represents 1-D turbulence where two of the components are smaller than the other. Finally, the origin represents isotropic turbulence, where all the components are of the same magnitude. This bounding region with the defined states is seen in figure 3.15

![Figure 3.15: Anisotropy map with possible turbulent states](image)
As was noted in the analysis of the turbulence intensities, we expect to see 1-D turbulence when the streamwise intensity is increasing and the spanwise and wall-normal intensity are decreasing. We note that the spanwise and wall-normal turbulence intensities reach a minimum around phase 7. Figure 3.16 shows the anisotropy map for phase 7, where each point represents the invariant values at a wall normal location from $0 \to h$ and the colors transition from dark blue at $z = 0$ to light green at $z = h$.

![Figure 3.16: Reciprocating channel velocity anisotropy map for $\phi = 7$. Period 30](image)

Following the previous bounding line definitions, we see that at phase 7 the flow in the near wall region is largely 1-D and as you move away from the wall the flow is in the cigar-like region sticking to the right hand boundary. This is exactly as we would have expected from the turbulent intensity figures 3.11 through 3.13. Now let us shift our focus to the timing of the bursting events that break up the streamwise streaks seen in phase 7. Previously we noted that the maximum values of spanwise and wall-normal fluctuations occur around phase 13, which also coincides with a max stokes’ layer streamwise fluctuation. We noted that at phase 16 the streamwise fluctuations had reached a minimum, leaving the
flow in a largely 2-D turbulent state. Let us look at the anisotropy map to verify the state of the turbulence at phase 16. Figure 3.17 shows the anisotropy map for phase 16 for the same wall-normal span and coloring scheme as 3.16.

![Figure 3.17: Reciprocating channel velocity anisotropy map for $\phi = 16$. Period 30](image)

From this plot it is clear that the turbulent state has shifted vastly from phase 7. The blue near wall region is now located in the 2-D pancake-like state and as you move further away from the wall you move from a pancake-like state to the cigar-like 2-D state, and then reach a region of isotropic 3D turbulence at the channel half-height. With this solid understanding of the turbulent reciprocating channel flow characteristics, we would like to explore more deeply the development of the wall shear stress and what factors contribute to its production.
3.7 **Wall Shear Integral Contributions**

Following the work of Fukagata (14), we derive an analogous relation for the case of the reciprocating channel flow. A quick summary of how this relation is derived is as follows, beginning with the simplified Reynolds’ Averaged Navier-Stokes equation for our reciprocating turbulent channel,

\[
\frac{\partial \overline{u}}{\partial t} = -\frac{dp}{dx} + \frac{\partial}{\partial z} \left( -\overline{u'w'} + \frac{1}{Re} \frac{\partial \overline{u}}{\partial z} \right) \tag{3.10}
\]

where,

\[
-\frac{dp}{dx} = A \cos(\omega t) \quad \text{with} \quad A = 1 \tag{3.11}
\]

Here we see the flow is driven by a cosinusoidal pressure gradient only in the streamwise direction. An over bar \( \overline{f} \) represents the average of a quantity \( f \) in the streamwise \((x)\) and spanwise \((y)\) direction

\[
f(x, y, z, t) = \overline{f}(x, y, z, t) + f'(x, y, z, t) \tag{3.12}
\]

If we integrate equation 3.10 three times in the wall normal direction up to the half channel height, as such,

\[
\int_0^h \int_0^z \int_0^z \frac{\partial \overline{u}}{\partial t} dz^3 = \int_0^h \int_0^z \int_0^z -\frac{dp}{dx} dz^3
\]

\[
+ \int_0^h \int_0^z \int_0^z \frac{\partial (-u'w')}{\partial z} dz^3 + \int_0^h \int_0^z \int_0^z \frac{\partial^2 \overline{u}}{\partial z^2} dz^3 \tag{3.13}
\]

we are left with the following rearranged relation for the wall shear stress,
\[ \tau_w = \frac{h \, dP}{3 \, dx} + \frac{2 \nu}{h^2} \int_0^h \tau_d \, dz + 2 \frac{h}{h^2} \int_0^h (h - z)(-\bar{u}'\bar{w}') \, dz - \frac{1}{h^2} \int_0^h (h - z)^2 \frac{\partial \bar{u}}{\partial \bar{t}} \, dz \] (3.14)

This relation shows the four terms that contribute to the wall shear stress for the turbulent case. The same relation is easily extended to the laminar case where the third term on the right hand side of equation 4.4 representing the turbulent fluctuations is non existent, for completeness, the equation is written as,

\[ \tau_w = -\frac{h \, dP}{3 \, dx} + \frac{2 \nu}{h^2} \int_0^h \tau_d \, dz - \frac{1}{h^2} \int_0^h (h - z)^2 \frac{\partial \bar{u}}{\partial \bar{t}} \, dz \] (3.15)

with

\[ \frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{dP}{dx} + \frac{1}{Re} \frac{\partial^2 \bar{u}}{\partial z^2} \] (3.16)

and the identical pressure gradient as equation 3.11.

To begin our analysis of these componential contributions, we will compare the sum of each term as function of the phase for DNS versus laminar. Figures 3.18 and 3.19 show the comparison of the mean velocity term and the difference between DNS and laminar, respectively.
Upon first comparison the turbulent case is nearly identical to the laminar case, with just a slightly larger contribution for the first half of the flow and slightly smaller contribution due to symmetry for the second half. Based on this small deviation in contribution it is clear that the presence of the stokes' layer peaks in the laminar case do not contribute much to the velocity term and the over and undershoot appear to cancel out in the overall
half-height integration. It is also noted that the velocity contribution profiles are about 90 degrees out of phase with the cosinusoidal pressure gradient. Referring to figure 3.19 we see that the max difference between the DNS and laminar case is over 200 times smaller than the max difference in wall shear stress, showing how small this term’s contribution truly is.

The second contributing term that can be directly compared is the transient term, which represents the contribution of the sum of the terms on the RHS of equation 3.10 for the turbulent case and 3.16 for the laminar case. The difference between which is the inclusion of the turbulent shear or Reynolds’ stress for the turbulent case. Figure 3.20 and 3.21 show the phase averaged contributions and difference between DNS and laminar, respectively.

![Graph](image)

*Figure 3.20: Phase averaged transient term comparing the DNS and laminar. Period 30*
Figure 3.21: Difference between the phase averaged transient term for DNS minus laminar. Period 30

Just as with the velocity term, we see very close agreement for the transient term between the turbulent and laminar case. However this close agreement is a larger surprise than the velocity result since the transient term for the turbulent case includes the extra gradient of Reynolds' shear stress. Based on the difference in figure 3.21, we see that the max difference is about one order of magnitude smaller than max wall shear difference.

Given these results it is concluded that the addition of the turbulent stresses are what produce the large deviations in the wall shear. Figure 3.22 shows the laminar and turbulent shear stress profiles and the laminar wall shear with the contribution of the turbulent shear stress added to it.
Figure 3.22: Comparison of laminar wall shear with and without turbulent contribution against DNS wall shear. Period 30

It is clear to see that the addition of the turbulent contribution to the laminar wall shear produces a profile that is close to the turbulent case. The deviations come from a combination of the transient and velocity terms which are also affected by the turbulent stresses. Figures 3.23 shows specifically the turbulent stress term. The most interesting feature of the

Figure 3.23: Contribution of turbulent stress to overall wall shear. Period 30
This figure gives us direct insight into the role that turbulence plays in this complex reciprocating flow. At the beginning of the flow period, the turbulent stress has a negative contribution to the wall shear stress, which weakens as the flow continues to accelerate to phase 8. As soon as the flow begins to decelerate at phase 9 we see an increase in turbulent contribution. Following this increase we reach a maximum contribution around phases 13 and 14 which represents the timing near when the flow is about to reverse direction. At phase 16 the turbulent contribution reaches a maximum rate of decrease, and with the now negative velocity the same but opposite process occurs.

In order for the flow to have a negative contribution the values of turbulent shear \((u'w')\) must be positive on average over the channel height. This is an important concept, since in a steady channel flow a positive average turbulent shear is not usual. It is of interest to understand what aspect of the flow is producing this positive turbulent stress and this is the subject of the following section.

### 3.8 Flow Topology

Based on the results of the integral analysis in section 3.7 we have determined that the main contribution to the variation in wall shear from the laminar to DNS is the addition of the Reynolds’ shear stress. In order to gain deeper insight on how and why the Reynolds’ shear influences the wall shear the way it does we will analyze the joint probability density function (JPDF) of the two important topological variables Q and R, which are the second and third invariants of the velocity gradient tensor.

Following the work of Chacin (4) we calculate these variables at each location in the flow field for each time and we produce a JPDF for each phase of the flow period. Along with the JPDF, at every given bin we will calculate the average value of Reynolds’ shear \((-u'w')\) to determine the topology of the flow where the positive and negative values of this stress are produced. Figure 3.24 shows an example JPDF with Reynolds’ stress for a
Figure 3.24: JPDF of \( Q-R \) with filled contour of \(-\overline{w'w'}\) for turbulent boundary layer from Chacin et. al.

It is believed that the production of the positive values of Reynolds’ shear occur within the stokes layer and the negative Reynolds’ shear is produced above the Stokes’ layer due to the inflection point found in the velocity profiles seen in figures 3.26, 3.30 and 3.32 where the inflection point is highlighted. We will focus on two specific regions for this analysis, above and below the inflectional point in the velocity profile, where the velocity gradient approaching zero. Referring back to figure 3.23, we will focus on three main phases, specifically, 1, 8, and 14, which encompass the most negative and positive contributions along with a phase with zero contribution, for the first half of the flow period. We begin by
looking at JPDFs for phase 1, starting with the region below the Stokes’ layer, which for phase 1 is \(0 \leq y/h \leq 0.093\). The JPDFs of \(Q\) and \(R\) contain contour lines of \(Q\) and \(R\) and filled contours representing the average value of \(-\overline{uw'}\) at each bin. The \(Q-R\) space is split into 40 bins, where the limits of \(Q\) and \(R\) are defined such that the average Reynolds’ shear is within 1\% of the statistical phase average. Figure 3.25 shows the JPDF for phase 1, and it is filtered such that only values of \(-\overline{uw'}\) are shown if the associated bins have a count of 10 or more. With this filter there is still 94.7\% of the average Reynolds’ shear calculated for phase 1. Figure 3.26 also shows the velocity profile for phase 1 for reference.

\[
\begin{align*}
-\overline{uw'} & = 0.0001718966074 \\
-\overline{uw'}\text{ DNS} & = 0.0001814410966 \\
\%\text{ DNS} & = 0.947396210789
\end{align*}
\]

Figure 3.25: JPDF of \(Q-R\) with filled contour of \(-\overline{uw'}\) for phase 1, filtered such that the count of a shown bin is above 10. Period 30
Figure 3.26: Velocity profile as a function z/h at phase 1 showing the location of the inflection point. Period 30.

It is clear from figure 3.25 that there is more production of positive Reynolds’ shear and it occurs mainly in quadrant IV of the Q-R phase space, representing a region near the edges of vortices. Comparing figure 3.25 with 3.24 we see a similar location of positive Reynolds’ stress production. It is also noted that quadrants I and II representing vortical structures, have some intermittent regions of negative Reynolds’ stress. If we now switch focus to the region above the Stokes’ layer for the same phase, which occurs after the inflection point of the velocity profile we gain insight into where the majority of the negative Reynolds’ stress must be produced to provide an overall negative contribution to the wall shear. Figure 3.27 shows the corresponding JPDF for the range 0.093 < y/h ≤ 1 again filtered for counts larger than 10.
It is clear from the size of the Q and R axes that this region of the flow contains stronger velocity gradient events as well as many regions of intermittent events. In order to better understand the structure, we will increase the filter to only show regions where the count is equal to 100 or more. Figure 3.28 shows this filtered JPDF for phase 1 above the stokes layer.
You can see that the value of Reynolds’ shear compared to the DNS has dropped by less than 1%, but now the overall characteristic is more clear. Above the Stokes’ layer we have a large production of negative Reynolds’ stress, which is produced mainly in quadrant III and IV, which is the same region as the positive production within the Stokes’ layer. It is important for this analysis to note the definition of turbulent production in the transport of Reynolds’ stress, defined as,

\[ P_{ij} = -\left( u_j u_m \frac{\partial u_i}{\partial x_m} + u_j u_m \frac{\partial u_i}{\partial x_m} \right) \]  

which for our case simplifies to,

\[ P = -u'w' \frac{\partial \overline{u}}{\partial z} \]  

Looking at phase 1 within the Stokes’ layer, we have positive values of Reynolds’ shear \(-u'w'\) and positive velocity gradients \(\partial \overline{u}/\partial z\), so together there is a positive production of
turbulence. Above the stokes layer, we have negative values of Reynolds’ shear, but also negative velocity gradients, so again they create a positive production of turbulence.

Continuing now to phase 8 where figure 3.23 shows roughly zero contribution from the Reynolds’ stress, we plot the same JPDFs. Figure 3.29 shows the JPDF at phase 8, within the Stokes’ layer, which for phase 8 is $0 \leq y/h \leq 0.29$ and filtered by a count greater than 100, again the corresponding velocity profile is shown for reference.

$$Q/Q_w = 0.00350594759396$$

$$-uw = 0.00362807883398$$

$$\% DNS = 0.966337214377$$

**Figure 3.29:** JPDF of $Q-R$ with $-uw'$ for phase 8 for $0 \leq y/h \leq 0.29$, filtered such that the count of a bin is above 100. Period 30
Here we see that filtering to a count of 100, we still maintain 96.6% of the average Reynolds’ shear. Again quadrant III and IV show strong levels of positive Reynolds’ shear. Quadrants I shows an intermittent region of positive Reynolds’ shear and quadrant II shows a small amount of negative Reynolds’ stress. Overall the space is mainly dominated by positive Reynolds’ stress. Noting the bounds of the Q-R space, the velocity gradients are small within the Stokes’ layer, meaning this stress is made by small fluctuating events. Let us now look at the results above the Stokes’ layer. Figure 3.31 shows the JPDF above the stokes’ layer at phase 8 filtered for counts of 100 or more.
Here we see that above the stokes layer there is little to no positive Reynolds’ stress and the production of negative Reynolds’ stress is large and mainly in quadrants I and IV. We note the span of the Q-R space is drastically larger than below the Stokes’ layer. However we know that at this phase, there is little contribution either positive or negative from the Reynolds’ stress to the wall shear, so although the structures may be larger in this region of the channel, they must be of same strength as the small scales found in the Stokes’ layer in order to produce no considerable contribution.

Lastly we will examine phase 14, where again from figure 3.23 we know we see the largest contribution to the wall shear from the turbulent stress. Since the velocity profile of phase 14 seen in figure 3.32 doesn’t exhibit any inflection point within the channel half-height, we perform the Q-R JPDF over the entire half-height in figure 3.33 where a filter of 1 is applied.
Figure 3.32: Velocity profile as a function $z/h$ at phase 14 showing the location of the inflection point. Period 30.

Figure 3.33: JPDF of $Q-R$ with $-\overline{uw}'$ for phase 14 for $0 \leq y/h \leq 1$, filtered such that the count of a bin is above 1. Period 30.

With a filter of 1, we are able to capture 99.9 % of the average Reynolds’ stress. Figure 3.33 exhibits exactly what we expected to see. The overall channel half-height produces
positive Reynolds' stress. Since there are still some visible intermittent events, we will look at the 10 and 100 filtered cases in figures 3.34 and 3.35.

Figure 3.34: JPDF of Q-R with $-u'w'$ for phase 14 for $0 \leq y/h \leq 1$, filtered such that the count of a bin is above 10. Period 30

Figure 3.35: JPDF of Q-R with $-u'w'$ for phase 14 for $0 \leq y/h \leq 1$, filtered such that the count of a bin is above 100. Period 30
As is expected increasing the filter, reinforces the structures and trends seen in 3.33. It is important to note the change in shape of the Q-R contours for this phase. We see the tail of the tear drop extends far into the (-) Q and (+) R region, which means that the flow structures are largely dissipative at this phase, which all makes sense since phase 14 is at the end of the deceleration period and the flow is about to reverse directions. It is known from (7) that during the deceleration phase the turbulent streaks found near phases 8 and 9, will become unsteady and burst apart into small scales fluctuations which are of the same size as the dissipative scales. It is believed that this is the reason that we see a trend towards this corner of quadrant IV. Given the results from this analysis we know have an idea of what flow structures are present in the flow as well as what kind of structures produces positive or negative Reynolds’ stress at varying phases. With this extensive analysis of turbulent and laminar reciprocating flows, we now move onto the second part of this thesis, which will analyze the use of RANS modeling to simulate the same reciprocating channel flow.
Chapter 4

RANS Modeling

4.1 Mean Flow Statistics

OpenFOAM computational fluid dynamics (CFD) was used to simulate a 2D complimentary Reynolds’ average Navier-Stokes’ (RANS) reciprocating channel simulation. Five common turbulence models available in the OpenFOAM package were utilized in the simulations. Three high-Reynolds’ models $k-\varepsilon$, $k-\omega$, $k-\omega$ SST and two low Reynolds’ models $k-\varepsilon$ Launder-Sharma, and $v^2-f$. We begin our analysis with the high Reynolds’ models. We first recognize that these models require the use of wall functions which assume a logarithmic velocity and temperature profile. As was shown in 3.2.1 figure 3.5 there is no presence of the standard log-law for period 30 and 3.7 only shows a couple phases that approach it, but overall the assumption of a logarithmic velocity law is not well defined for this type of flow. Since this research is also related to engine simulation it is important to look at the predictions of temperature transport by these RANS models, we specifically look at temperature as passive scalar to keep the simulations’ costs low and simple.

We will begin by focusing on the velocity and temperature predictions of the high-Reynolds’ models. Figure 4.1 shows velocity predictions of the Hi-Reynolds’ models for the first half of the flow versus the direct numerical simulation (DNS) results and figure 4.2
show temperature predictions for a few phases of the flow compared with the DNS.

Figure 4.1: Average velocity profile comparison as a function of $z/h$ for the high-Reynolds’ models versus DNS. Figure A. $\phi = 1 - 8$ and figure B. $\phi = 9 - 16$. Period 30

Figure 4.2: Average temperature profile comparison as a function of $z/h$ for the high-Reynolds’ models versus DNS for various phases. Period 30

The Hi-Reynolds’ models produce accurate core velocity predictions for most phases, but are severely lacking in accuracy near the inflection point, which is expected because
of the low grid resolution and assumption of logarithmic near wall profile. The temperature predictions are less accurate across the channel height than the velocity predictions. However overall it is safe to say that the high Reynolds’ models are doing a decent job in predicting mean quantities. We believe however that if the wall functions were designed for this non-equilibrium flow we would see an improvement in the predictions. We would like to acknowledge that there have been non-equilibrium wall models (27)(5) developed that could be tested, however this is outside of the scope of this project and is considered continuing research.

To remove the reliance on the wall functions, our next analysis looks into the improvements that can be made in terms of predictions by fully resolving the near wall layer and applying two common low Reynolds’ turbulence models. As was stated previously we are using the formulations of $k - \varepsilon$ Launder-Sharma and $v^2 - f$ implemented in OpenFOAM. The same velocity and temperature profiles are shown in figures 4.3 and 4.4 for these low Reynolds’ models.

![Figure 4.3: Average velocity profile comparison as a function of $z/h$ for the low-Reynolds’ models versus DNS. Figure A. $\phi = 1 - 8$ and figure B. $\phi = 9 - 16$. Period 30](image)

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In the velocity profiles we see that the full near wall resolution vastly improves the overall predictions. There is still a small deviation near the inflection points, but they are much improved versus the Hi-Reynolds’ models. Surprisingly, not much of an improvement is found for the low Reynolds’ temperature predictions. We see the profiles are under predicted in the same way the high Reynolds’ models were. Although these models appear to predict mean flow quantities with an acceptable level of error, for many applications the prediction of near wall metrics is equally or more important. The next section will look at two important near wall metrics related to the momentum and thermal fields.

4.1.1 Near-Wall Predictions

An important metric for the study of turbulent flows is the wall shear stress $\tau_w$, which can be used to determine how well a model can predict near wall flows. The wall shear stress is defined as,
Figure 4.5: Phase averaged wall shear stress profile for high-Reynolds’ models versus DNS. Period

The High Re models vastly over predict the wall shear near the peak velocity through the late deceleration of the first half of the flow and the same is seen for the second half of the flow. There is good agreement for the first 3 phases and the final 4 phases of the half period. This over prediction shows that the model is predicting far more turbulence than the DNS, from our understanding of wall shear production from section 3.1. Along with the wall shear stress, we would like to analyze the models ability to accurately predict the near wall heat transfer or the Nusselt number. The Nusselt number is defined as follows,
where $\bar{\theta}_h = 1$ $\bar{\theta}_c = 0$ and $H = 2$. As was mentioned previously our code only simulates temperature as a transported scalar so buoyant affects are neglected, however, one can assume that if a model fails at simulating scalar transport, it cannot perform better when adding another level of complexity. Figure 4.6 shows the phase averaged Nusselt number for the three high Reynolds’ models.

![Phase averaged Nusselt number for high-Reynolds’ models versus DNS. Period 30](image)

Figure 4.6: Phase averaged Nusselt number for high-Reynolds’ models versus DNS. Period 30

Figure 4.6 shows the predictions of the high Reynolds’ models are devastatingly incorrect with the Nusselt number massively under predicted and out of phase at all times and lacking any of the features seen in the DNS profiles. In respect to the use of these models in simulating non-equilibrium thermal fields, one should be cautious in accepting any solution produced by these models. After seeing the near-wall predictions of the high Reynolds’ models, let us now analyze the same metrics for the low Reynolds’ models and determine what the inclusion of the full near wall layer does in regards to improving accuracy. Figure 4.7 shows the predictions of wall shear for the low Reynolds’ models.
With the inclusion of near-wall resolution there is a large increase in prediction accuracy for the wall shear stress. We see good prediction for all but 5 phases from phase 9 through 12. The largest difference between the high and low models is the low models' ability to accurately maintain the correct turbulence levels during the accelerating period from phase 1 to 8. As seen in figure 4.5 the high Reynolds' models largely over predict the turbulence levels as soon as phase 3. The low Reynolds' models are still not without error though, as they appear to over predict the timing and magnitude of the turbulence around phase 9. Next let us see what improvements are made in prediction of the near-wall heat transfer. Figure 4.8 shows the phase averaged Nusselt number for the low Reynolds' models.
The results of the low Reynolds’ models for the Nusselt number prediction are much better than the former high Reynolds’ models. The shape of the DNS profile is very well predicted, with a few main errors. There is a clear phase advance for the low Reynolds’ models, which produces an early decrease in Nusselt during the acceleration period and an early increase in Nusselt during the deceleration period. The final error is the large spike in Nusselt number seen only for the Launder-Sharma model. The improved results from the full near-wall resolution are not unexpected, since both metrics rely on an accurate prediction of velocity and temperature gradients at the wall one would expect the full near wall resolution to provide more accurate predictions. Still of question to us is whether the error in the wall shear stress near phases 9-12 is directly caused by poor turbulent production prediction and where the phase advance and spike in the Nusselt number come from. To begin analyzing these questions in the following section we will perform the same integral analysis as in section 3.7 on the RANS data.
4.2 Integral Contributions to Wall Shear Stress

Again following the work of section 3.7 we would like to investigate the contributions to the wall shear stress based on the RANS data. For this section we will only be focusing on the low Reynolds’ models, however this same analysis can be applied to the high Reynolds’ models as well. As was seen in section 3.7 the integral relation for the RANS modeling is the same as 4.4, except the Reynolds’ stress term is replaced by the RANS modeling equivalent,

$$-\overline{w'w'} = \nu_T \frac{\partial \overline{u}}{\partial z} \quad (4.3)$$

which gives us the following integral relationship for the RANS wall shear stress,

$$\tau_w = \frac{-h}{3} \frac{dP}{dx} + \frac{2\nu}{h^2} \int_0^h \overline{u} dz + \frac{2}{h^2} \int_0^h (h - z)(\nu_T \frac{\partial \overline{u}}{\partial z}) dz - \frac{1}{h^2} \int_0^h (h - z)^2 \frac{\partial \overline{u}}{\partial t} dz \quad (4.4)$$

With this equation it is possible to analyze each term on the RHS as was done previously to gain insight into where the RANS modeling is falling short compared to the DNS. To begin, we will name for convenience the terms of equation 4.4 as was done in section 3.7. Starting on the RHS of the equations, we label the terms in order, $I$ is the pressure gradient term, $II$ is the mean velocity term, $III$ is the turbulent heat flux term, and $IV$ is the transient term. Given this naming convention, let us now compare the terms for the RANS and DNS results. Figures 4.9 through 4.15 show each term averaged over each phase. As we did in section 3.7 we will begin by looking at the phase averaged contribution of the velocity term and the difference between the RANS and DNS velocity term.
The velocity term for the RANS models shows very little deviation from the DNS results. This is an interesting result, since there is visible deviation in many of the velocity profiles.
In the end however the contribution of the velocity term to the wall shear stress are small. Figure 4.10 shows the difference between DNS and each RANS model. We see the max difference in velocity contribution is around 0.00012 (0.4%) and 0.0001 (0.7%) for the LS and $v^2 - f$ models respectively, showing just how little an error in velocity profiles affects the wall shear. Next we compare the transient terms in figures 4.11 and 4.12.

![Figure 4.11: Phase averaged transient term comparing low-Reynolds’ models and DNS. Period 30](image-url)
From these figures we see that from a direct comparison of the transient terms both RANS models appear to do well in predicting the results of the DNS, however, upon closer inspection of the difference between the RANS and DNS we see that there is significant differences in the phases 8-16, which is believed to be caused mainly by the gradient of the turbulent term within the transient term. We see the max difference for the transient term compared to the max wall shear difference, the transient term makes up around 65% and 84% of the wall shear max difference for LS and $v^2 - f$ respectively. Next we will look at the Reynolds’ stress term in figures 4.13 and 4.14.
Here it is clear that there are large visible deviations of the RANS models compared to the DNS. The deviations in figure 4.13 appear to be easily recognized in the wall shear
comparison of figure 4.7. The early increase seen in the wall shear matches the early increase in the Reynolds’ stress term. Comparing the max difference of the Reynolds’ stress term with the max wall shear difference, we have 45% and 35% contributions from LS and \( v^2 - f \) respectively. Finally we will compare the pressure gradient term for the RANS and DNS in figures 4.15 and 4.16.

![Figure 4.15: Phase Averaged \( dp/dx \) term comparing RANS and DNS. Period 30](image-url)
We see as is expected the direct comparison of the RANS and DNS pressure terms are very close. The slight discrepancy is due to the computational mesh for the RANS cases is lacking a centrally located cell. This causes the integration to be performed from 0 to 1.0085, so the pressure gradient terms has \( h = 1.0085 \) instead of \( h = 1 \) for the DNS.

Now that we have compared the individual terms for the RANS versus DNS data it is known that the velocity term is the smallest contributor to the wall shear stress and the terms which involve the Reynolds’ stress have the largest contributions. Referring back to the transient and pressure gradient terms, we notice that they appear to be very close in shape and only vary by a minus sign, in order to more deeply understand these profiles, figure 4.17 shows the addition of the two terms for the DNS and RANS models.
Figure 4.17: Phase averaged transient term + dp/dx term comparing DNS and low-Reynolds’ models. Period 30

The combination of terms $I$ and $IV$ provides an explanation for part of the error found in the wall shear stress profiles. Around phases 8-10 both RANS there is a plateau and the models deviate from the DNS. It is also noted that this summation of $I$ and $IV$ is close to the laminar wall shear profile, as seen in figure 4.18
The main finding from the integral analysis is the understanding that the terms that contain the Reynolds’ stress, terms III and IV are the largest contributors to the overall shear stress profile and as such any errors in the prediction of Reynolds’ shear largely affect the accuracy of the wall shear stress prediction. We will take one more step in this analysis, specifically looking at terms III and IV and their cumulative contribution, or how the contribution changes over the channel half height. Integrating from 0 to $z$ instead of 0 to $h$ and plotting against the channel half-height will exhibit specifically where the models and DNS differ as the integration progresses. Starting with term IV, which again represents the transient term for the DNS and models, we can visualize the deviations at key phases. Figure 4.19 shows these plots at key phases of the flow period 30.
As is visualized in figure 4.12 phases 8-16 show a large deviation in the RANS transient term, so figure 4.19 shows a selection of phases within this range. From the cumulative profiles, we see that for all the phases shown, the only large deviations occur in the near wall region and they are mainly. Most notably the Launder-Sharma model shows large deviations in the near wall region, whereas the $v^2 - f$ is more accurate in the near wall region. Both models appear to match the shape of the DNS profile away from the wall, just offset due to the changes in the near wall values. We will now look at the cumulative contributions for term III, the Reynolds’ stress term, in figure 4.20
Shown in figure 4.14 the largest deviations of the first half of the flow period occur from phases 8-13, so we have investigated the cumulative contribution of the Reynolds’ stress term during those phases. Here we see both RANS models are over predicting the turbulent contributions mainly in the near wall region. The Launder-Sharma model shows very steep increases in contribution in the near wall region with an artificial looking slope and many phases showing a sharp transition. The $v^2 - f$ model on the other hand does not experience this artificial near wall slope, which we attribute to the lack of the damping function in the turbulence model. Based on these profiles there is a clear phase advance for both RANS models and poorly predicted near wall characteristics. Based on the finding of 4.2 it is clear that the simulation of this non-equilibrium flow requires the use of a new turbulence model or the modification of current models to accurately predict the near wall flow regime. Given the knowledge of the failure of the turbulent eddy viscosity closure, it is important to understand how the turbulent errors manifest themselves in the prediction.
of other flow quantities, such as the prediction of temperature. The following section will perform the same integral contribution analysis using the temperature transport equation to derive an equation for the contributing terms to the wall heat flux.

4.3 Integral Contribution to Wall Heat Flux

As the final part of this research, we are interested in determining the contributions to the near wall heat flux or Nusselt number, which we have shown in 4.1.1 is poorly predicted using the high Reynolds’ models and has improved prediction when using the low Reynolds’ models. However since we have determined in 4.2 that there is an error within the modeling of the turbulent stress, it is important to determine whether the errors in Nusselt number prediction are fragments of other errors, or directly related to the turbulent closure required in the thermal transport equation. To begin this analysis we will look at the scalar temperature transport equation stated as,

\[
\frac{\partial \theta}{\partial t} + \bar{u}_i \frac{\partial \theta}{\partial x_i} = \frac{\partial}{\partial x_i} \left( -\theta' \bar{u}' + \alpha \frac{\partial \theta}{\partial x_i} \right)
\]  (4.5)

which simplifies in the periodic channel case to,

\[
\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z} \left( \bar{w}' \theta' \right) + \alpha \frac{\partial^2 \theta}{\partial z^2}
\]  (4.6)

and with the RANS turbulent heat flux closure substituted in,

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left( \alpha_T \frac{\partial \theta}{\partial z} \right) + \alpha \frac{\partial^2 \theta}{\partial z^2}
\]  (4.7)

where the turbulent heat flux closure is as follows,

\[
\bar{w}' \theta' = -\alpha_T \frac{\partial \theta}{\partial z} \quad \text{with} \quad \alpha_T = \frac{\nu_T}{Pr_T}
\]  (4.8)
Following the work of (12) we integrate over the channel height three times,

\[ \int_0^h \int_0^z \int_0^z \frac{\partial \theta}{\partial t} (dz)^3 = \int_0^h \int_0^z \int_0^z \frac{\partial}{\partial z} \left( \frac{w' \theta'}{\bar{\theta}} \right) (dz)^3 \]

\[ + \int_0^h \int_0^z \int_0^z \alpha \frac{\partial^2 \theta}{\partial z^2} (dz)^3 \]  

(4.9)

which simplifies to,

\[ \frac{1}{2} \int_0^h (h-z)^2 \frac{\partial \bar{\theta}}{\partial t} dz = \int_0^h (h-z)(-\bar{w}' \bar{\theta}') dz \]

\[ + \frac{1}{RePr} \int_0^h \bar{\theta} dz - \frac{h}{RePr} \bar{\theta} \bigg|_0 - \frac{h}{2RePr} \frac{\partial \bar{\theta}}{\partial z} \bigg|_0 \]  

(4.10)

and for the RANS equation,

\[ \frac{1}{2} \int_0^h (h-z)^2 \frac{\partial \bar{\theta}}{\partial t} dz = \int_0^h (h-z) \left( \alpha_T \frac{\partial \bar{\theta}}{\partial z} \right) dz \]

\[ + \frac{1}{RePr} \int_0^h \bar{\theta} dz - \frac{h}{RePr} \bar{\theta} \bigg|_0 - \frac{h^2}{2RePr} \frac{\partial \bar{\theta}}{\partial z} \bigg|_0 \]  

(4.11)

Since we are interested in the contribution to the heat transfer at the wall or Nusselt number we rearrange this equation to determine the contributions of each term on the value of \( Nu = -2 \frac{\partial \bar{\theta}}{\partial z} \bigg|_0 \).

\[ Nu = \frac{4RePr}{h} \int_0^h (h-z)(-\bar{w}' \bar{\theta}') dz + \frac{4}{h} \int_0^h \frac{\bar{\theta} dz}{t} - 4 \frac{\bar{\theta}}{t} \bigg|_0 - \frac{2RePr}{h} \int_0^h (h-z)^2 \frac{\partial \bar{\theta}}{\partial t} dz \]  

(4.12)

and for the RANS equation,
\[
Nu = \frac{4RePr}{h} \int_0^h (h - z) \left( \alpha T \frac{\partial \theta}{\partial z} \right) dz + \frac{4}{h} \int_0^h \bar{\theta} dz - 4 \bar{\theta} \bigg|_0^h - \frac{2RePr}{h} \int_0^h (h - z)^2 \frac{\partial \bar{\theta}}{\partial t} dz
\]

(4.13)

As was done in section 4.2, we will name the terms of equations 4.12 and 4.13 for convenience. Starting on the right hand side of both equations, we will call the turbulent heat flux term \(I\), the integral of mean temperature term \(II\), the bottom wall temperature term \(III\), and the transient temperature term \(IV\). Again as before, we can plot the phase averaged values of these terms to see their overall contribution to the Nusselt number at each phase. We start with the less important terms, figure 4.21 shows the phase averaged contributions of the wall temperature on the Nusselt number contribution.

Figure 4.21: Phase averaged contribution of term III comparing low-Reynolds' models and DNS. Period 30

Due to the identical boundary conditions at the wall the contribution is the same for both the DNS and RANS. This term is only important for its addition to the Nusselt
number. Now we look at the contribution of the mean temperature in figure 4.22.

![Figure 4.22: Phase averaged contribution of term II comparing low-Reynolds' models and DNS. Period 30](image)

Here we see that there is some slight deviation between the RANS models and the DNS. However the contributions of the deviations are small compared to the overall value of Nusselt number, roughly, 5% and 6% for the LS and $v^2 - f$ models respectively. Now let us focus on the terms that provide the bulk of the contribution towards the Nusselt number. Figure 4.23 shows the contribution of the turbulent heat flux term.
Here we see the Launder-Sharma model shows a moderate prediction of the turbulent contribution for the first 9 phases being slightly under predicted. The $v^2 - f$ shows much poorer prediction for the first 5 phases, however it improves over LS at phases 6-10. Following phase 10 both models under predict the timing of the increase as well as the magnitude of the increase in turbulent contribution. Looking at the LS model, we do see the appearance of an early increase in turbulent contribution around phase 8, which is the same phase at which the early increase occurs in the Nusselt number. However the $v^2 - f$ model appears to match the DNS turbulent contribution well during the phases when the $v^2 - f$ Nusselt number shows an early increase. The final term to analyze is the transient term seen in figure 4.24.
The transient term shows the largest disparity of any of the terms, with the RANS models being completely out of phase with the DNS. Again we see the presence of the early increase in the LS model near phase 8. For the $v^2 - f$ model we also see the emergence of the early increase which explains the corresponding increase in the Nusselt number. As we know from the analysis of terms $II$ and $III$, the terms $I$ and $IV$ must produce the majority of the Nusselt number profile. Figure 4.25 shows terms $I$ and $IV$ added together to verify this.
With the combination of terms $I$ and $IV$ we recover the overall shape of the Nusselt number profiles for both RANS and DNS models. This means the total Nusselt number is mainly affected by the turbulent term, which appears in both terms $I$ and $IV$, and the second derivative of the temperature. Since it is known that the turbulent viscosity is poorly predicted by both RANS models, it is not a surprise that the turbulent heat flux terms are also poorly predicted since it is directly related to the eddy viscosity. For completeness, we will look at the cumulative contributions for terms $I$ and $IV$ at a few select phases. Figure 4.26 shows the cumulative contribution of term $IV$ as a function of channel height.
The cumulative contributions of the transient term, show the same near wall deviations like those seen in section 4.2. The LS model breaks down in the near wall as seen in each of the phases, and that poor prediction affects the final value of the contribution. The $v^2 - f$ model is more well behaved near the wall compared to LS, however it is still plagued by some near wall deviation. Comparing the qualitative shape of the $v^2 - f$ profiles, we see that it appears to capture the shape of the DNS. Figure 4.27 shows the cumulative contribution of the turbulent heat flux term $I$. 
Figure 4.27: Cumulative contribution of term I as a function of channel half height, phases 7-12.

Period 30

Here we see the cumulative contribution of the turbulent heat flux is more well behaved than the previous term. The LS model still shows artificial features in the near wall region causing an increase. The \( v^2 - f \) model predicts quantities much closer to the DNS and lacks any near wall deviations. Comparing the shape of the profiles from \( y/h > 0.5 \) we see good prediction of the shape compared to the DNS.

4.4 REYNOLDS’ ANALOGY

Although it is clear that the main contributing factor to the errors in Nusselt number prediction is the turbulent heat flux closure, it is also important to briefly mention the other assumptions that are included in the heat flux closure. The main assumption used to relate the eddy viscosity to the turbulent heat flux is the Reynolds’ analogy. The Reynolds’ analogy relies on the assumption that the turbulent Prandtl number \( Pr_T \) is constant at
every location within a flow. This assumption holds relatively true in steady equilibrium
flows, but breaks down in unsteady non-equilibrium flows.

We first will look at the results from a DNS of steady periodic turbulent channel flow,
with isothermal walls and passive transport of temperature. We can define for the DNS
simulation, the exact definition of the turbulent Prandtl number is,

\[ Pr_T = \frac{\nu_T}{\alpha_T} \]  

(4.14)

where

\[ \nu_T = \frac{\bar{u'}w'}{\partial \theta/\partial z} \quad \alpha_T = \frac{\bar{w'\theta'}}{\partial \theta/\partial z} \]  

(4.15)

written exactly for the DNS we have,

\[ Pr_T = \frac{\bar{u'}w' (\partial \theta/\partial z)}{\bar{w'\theta'}(\partial \pi/\partial z)} \]  

(4.16)

This exact definition is plotted as a function of the channel height for the steady channel
flow in figure 4.28
It is easy to see that the turbulent Prandtl number can be considered about constant across the channel height for the two different Reynolds’ numbers studied. From the same study, we can compare the passive temperature with two buoyant cases, with two different buoyant intensities defined by the Richardson number $Ri_T$. The turbulent Prandtl numbers for these cases are compared with the passive case in figure 4.29.
With the addition of buoyant forces the constant turbulent Prandtl number assumption fails to hold, as the value drops steeply for both cases as you move towards the channel center.

Using the DNS data of our reciprocating turbulent channel flow, we calculate the same exact turbulent Prandtl number and determine whether the assumption still holds. For sake of cleanliness, the $Pr_T$ number will only be plotted for a few different phases within the flow period. The $Pr_T$ number is plotted for phases 1, 5, 9, and 13 in figure 4.30.
Here we see that for the reciprocating channel the $Pr_T$ number varies by a large amount, both across the channel height, as well as between different phases of the flow. It is recognized that due to the inflection point in the velocity profiles caused by the stokes layer flow, the $Pr_T$ number has locations where the value goes to infinity because the velocity gradient approaches zero at the inflection point. These cases help to show both ends of the spectrum of applicability of the Reynolds’ analogy, it is applicable for a steady channel, and highly non-applicable for the buoyant and reciprocating channel flow. One must imagine that any flow with complex features of the levels found in the buoyant and reciprocating channel will exhibit the same lack of applicability for the Reynolds’ analogy. Relating to the errors in the Nusselt number it is most likely that they are due to a combination of the eddy viscosity and the Reynolds’ analogy, either way a new model or modification to the current models should be investigated.
4.5 **Strength of Integral Methods**

As a final section we would like to show the ultimate strength of the integral methods that were derived and applied previously. Just for a recap, figure 4.31 shows a side by side comparison of the wall shear stress and the Nusselt number.

![Phase Averaged Wall Shear Stress](image1.png)  ![Phase Averaged Nusselt Number](image2.png)

*Figure 4.31: Phase averaged wall shear stress and Nusselt number. Period 30*

The standard method used in RANS model validation and performance assessment is the comparison of statistical quantities predicted by RANS to experimental or DNS data. Such quantities include wall shear stress, mean velocity profiles, wall heat flux, etc. Here, the low-Reynolds’ RANS models produce decent overall predictions of the wall shear and Nusselt number. There are still clearly areas of discrepancy for both terms, which has been the focus of much of this research. The use of the integral methods, allowed us to determine the contributions of different terms to both near wall metrics, again we can see these terms in figures 4.32 and 4.33 for the wall shear and Nusselt number respectively.
Figure 4.32: Wall shear stress contributing terms, I is the pressure gradient term, II is the mean velocity term, III is the Reynolds’ stress term, and IV is the transient term. Period 30
Figure 4.33: Nusselt number contributing terms, I is the turbulent heat flux term, II is the mean temperature term, III is the wall temperature term, and IV is the transient term. Period 30

From these plots we are able to again compare the contributions of the different terms between the different models and DNS data. As is expected from the side by side comparison of wall shear and Nusselt number, the wall shear terms involved are much better predicted by the RANS models than the Nusselt number terms. This qualitative comparison between the models lends some insight into the predictive abilities, but our integral method also allows for the relative comparison of each term, by dividing each term by the wall shear and Nusselt number respectively. This comparison is seen in figures 4.34 and 4.35.
Figure 4.34: Relative contribution of wall shear stress terms, scaled to remove the asymptotic behavior. I is the pressure gradient term, II is the mean velocity term, III is the Reynolds’ stress term, and IV is the transient term. Period 30
Figure 4.35: Relative contribution of Nusselt number terms. I is the turbulent heat flux term, II is the mean temperature term, III is the wall temperature term, and IV is the transient term. Period 30

The advantage to this comparison is that instead of comparing how well the RANS models predict the DNS profiles, we can determine how well the RANS models predict the physics of the flow. In our study, a model that is a good predictor of the flow dynamics can be expected to show similar variation in time (or phase) and magnitude in relative contribution as the DNS data. Note, since we have divided the wall shear terms by the wall shear itself, we see spikes where the wall shear value approaches zero, however ahead and behind that point we can gain important insight into the models’ performance. Figure 4.34 shows that the largest deviations occur in term II or the contribution of the mean velocity profiles. Around phase 8, both RANS models predict a large drop in this term followed by an increase near phase 10. The relative contribution of I, III, and IV predicted by RANS are much closer to those of DNS. Interestingly the relative contribution of the turbulent stress is in close agreement with the DNS, which was unexpected given the profile from figure...
4.32. If we now look to the relative contribution of the Nusselt number terms in figure 4.35 we see a drastically different result. The relative contribution of II and III predicted by RANS show fair prediction however there is still a clear phase advance for both and some disagreement in overall magnitude. The most surprising result of this analysis is the comparison of the turbulent heat flux term I and the transient term IV. The RANS models are almost 180 degrees out of phase with the DNS results, but not surprisingly, in phase with each other. The main conclusion from this is that the closure of the turbulent heat flux, which is derived from the Reynolds’ analogy, is poorly predicting the physics of the thermal transport. However, since we saw that the relative contribution of the turbulent stress was well captured by RANS models in figure 4.34, the phase opposition of the relative contribution of the turbulent heat flux stems from the failure of the Reynolds’ analogy for this non-equilibrium flow. This analysis speaks to the use of these integral methods and their ability to provide a deeper insight than the common statistical analysis.
Chapter 5

Conclusion

5.1 Discussion

Our research has been motivated by the errors associated with applying equilibrium based turbulence models to the simulation of internal combustion engines and their effects on overall engine design. Currently, the design of internal combustion engines (ICEs) opts for the ease of computation and short turn over time for design cycles versus accurate time consuming resolved simulations. The use of coarse and unresolved simulation in the design process requires the use of prototyping in order to verify the proposed engine designs from the simulations. Given a more appropriate turbulence model for the non-equilibrium flows within an ICE, the reliance on prototypes for validation can be decreased and overall engine design cycle time decreases. This research has been presented in two main sections, one which fundamentally analyzes the difference between laminar and turbulent reciprocating flows and the other which compares different Reynolds’ average Navier-Stokes’ (RANS) turbulence models ability to simulate reciprocating flows, which represent some of the non-equilibrium physics within an ICE. The research also culminates in the introduction of an integral analysis validation method that is shown to be a strong addition to the standard validation technique.
5.1.1 **Laminar Reciprocating Channel**

From our analysis of laminar and turbulent reciprocating channel flows, it was determined that the presence of near wall turbulence drastically decreases the apparent size of the plug-like flow region, as the turbulent eddies transfer momentum away from the near wall and into the core flow. This is clear in the direct comparison of laminar and turbulent velocity profiles in figure 3.4. Given the prevalent use of wall models in industry it was important to investigate the classic logarithmic velocity law plots in figures 3.5 and 3.7 where we determined for period 30, no profiles show a log law and period 40 shows only a couple phases that approach the law. This was an important finding in regards to the use of the basic wall functions. Following the mean statistics, we investigated the isotropy of the flow by analyzing the invariants of the anisotropy tensor. From this it was determined that as the flow accelerates towards phase 8 it becomes more and more one dimensional as the near wall shear fluctuations produce quasi streamwise vortices and streaks seen in figure 3.16. The one dimensionality of the flow is verified by the mean statistics of the fluctuating components. The decelerating period of the flow becomes largely two dimensional in the near-wall region as the streamwise streaks oscillate and break-up in a bursting manor.

Next a direct comparison of the wall shear stress was performed, which gave an important look into how turbulence effects a reciprocating flow. The direct comparison of wall shear stress is seen in figure 3.1 and it was determined that during the early accelerating period of the flow from phases 1-6, there is less shear stress in the turbulent case. When the flow reaches peak forward velocity and has developed the streaks as previously mentioned, the shear stress accordingly increases over the laminar case. during the deceleration period near phases 13-14 we see the magnitude of the shear stress slope increases as the near-wall streaks oscillate and break apart. In order to verify that the turbulence was the main cause
of the variation in wall shear stress, we developed an integral relation, which examines the contributing terms to the wall shear stress. Based on the results from this analysis, it was verified that the main contributing term to the variation in wall shear was in fact the turbulent Reynolds’ stress. Figure 3.22 shows an important comparison, where the turbulent contribution is added to the laminar wall shear and we see close agreement between the DNS shear, again exhibiting that the turbulent term is the largest contributor.

The most intriguing feature of the turbulent reciprocating flow was the overall negative contribution of Reynolds’ stress during the accelerating period. Since an overall negative contribution is not commonly found in typical flow scenarios, we wanted to determine through a flow topology analysis, which features were producing the negative Reynolds’ stress and how it was related to the common positive Reynolds’ stress production in a turbulent channel. We determined that the inflection point within the velocity profiles are what cause the production of negative Reynolds’ stress, and move fluid packets from a high speed fluid near the wall into a region of slower speed further from the wall.

Using a joint probability density function (JPDF) we were able to visualize the Reynolds’ stress production on top of a Q-R map representing regions of high rotation and strain respectively. The JPDFs were analyzed at phases 1, 8 and 14, representing three unique phases within the Reynolds’ stress contribution, that being negative, zero, and positive contributions respectively. For phases 1 and 8 we performed a JPDF above and below the inflection point, and for 14 since no inflection occurs the entire half channel was included. From this analysis in phase 1 it was confirmed that negative Reynolds’ stress is produced above the inflection point, and it is formed by the same topology found in a turbulent flow, as is seen by comparing figures 3.25 and 3.28 with figure 3.24 from the work of Chacin et. al. (4). Phase 8 shows the same topology for the positive Reynolds’ stress production below the inflection point as seen in Chacin, however the negative Reynolds’ stress produced above the inflection show production in both quadrant IV and I, which makes sense since phase
8 contains more turbulent vortices represented by the quadrant I regime. Finally, phase 14 contains little to no negative Reynolds’ stress and shows production by the same topology as Chacin. The overall understanding from this analysis is that within a reciprocating flow the flow topology is very much alike a turbulent channel and the negative stress is only caused by the inflectional velocity profile. Interesting as well, since the velocity gradient is negative when the Reynolds’ stress is negative, there is still positive turbulence production.

5.1.2 RANS Reciprocating Channel

Following the laminar and direct numerical simulation (DNS) comparison, we compared some common high and low Reynolds’ number turbulence models to determine their accuracy in predicting non-equilibrium flows. To begin we performed this analysis using the standard validation technique of comparing mean velocity profiles and near wall metrics. Results for the high Reynolds’ models determined that their predictive capabilities are decent for the core flow velocity and temperature profiles seen in figures 4.1 and 4.2, however the reliance on wall functions produces large variations in the wall shear stress and Nusselt number seen in figures 4.5 and 4.6. It is possible that with a non-equilibrium wall function these models may better predict the near-wall phenomena, but at the moment this is not commonplace in industry.

With the high Reynolds’ model results seen we were curious to see what improvements are made using full near wall resolution and low Reynolds’ models. For both \( k – \varepsilon \) Launder-Sharma and \( v^2 – f \) we saw some slight improvements in core flow predictions over the high Reynolds’ models for velocity in figure 4.3, however the temperature predictions showed very little improvement, figure 4.4. Since these models remove the reliance on near-wall modeling, we saw large improvements in the predictions of wall shear stress and Nusselt number since the near wall gradients are fully resolved figure 4.7 and 4.8. Although a large
improvement was seen in near wall predictions, there were still phases where the models showed variations that could be detrimental to accurate simulations of ICEs.

Given the results of the standard validation techniques, we wished to gain a deeper understanding of where and why the models are breaking down. Performing the same integral analysis as was done with the laminar and DNS data, we applied our integral relations for wall shear stress to the RANS data and were able to compare both the overall contribution of different terms to the near wall metrics as well as the relative contribution of those terms. It was determined that the turbulent shear stress term showed the largest deviations in wall shear contribution, but the mean velocity term showed the largest deviation in relative contributions.

The same integral analysis technique was applied to the Nusselt number and again it was determined that the largest contributing terms to the Nusselt number were the turbulent heat flux and the transient term. However from the relative contribution analysis we found that the relative contribution of the RANS models’ turbulent and transient terms showed results that were 180 degrees out of phase with the DNS. Since the relative contribution of the turbulent shear stress was well behaved compared to the DNS, we believe that the use of the Reynolds’ analogy is the largest contributor to the errors in the Nusselt number. Since the Reynolds’ analogy relates to the physical relation of the momentum to the thermal transport, it is not surprising that the out of phase results seen in terms I and IV in figure 4.35 would be caused by the breakdown of this physical analogy.

5.2 CONTRIBUTION

In the end of this research we have shown that the RANS simulation of reciprocating flows using both high and low Reynolds’ models can produce results that are deemed acceptable for velocity predictions, but the thermal predictions present themselves to be much less
accurate and overall improvements to the models should be made. During the course of these comparisons, we have also determined a new technique for model validation, which we believe should be included with the common techniques used in model research. As such, we suggest the following new procedure for future model validation. Given a benchmark simulation or experiment, the complimentary RANS simulation is performed, the results should then be compared in the standard way, by comparing mean profiles and near wall metrics directly. Following the standard procedure it is advised that the integral analysis method applied in sections 4.2 and 4.3 be used in order to determine the contributions of different flow terms on the near wall metrics. With this more rigorous evaluation added to the standard technique, RANS models may be validated on a more in depth level and great improvements can be made to new or existing models.

5.3 Future Work/Research

With the knowledge that the eddy viscosity model used in the Reynolds’ stress closure is the largest contributor to errors in the wall shear stress and the Reynolds’ analogy for relating the momentum and thermal fields produces the largest errors in the Nusselt number. The future work will focus on the optimization of the \( v^2 - f \) model as well as the implementation of new turbulence models in OpenFOAM that may be better suited for simulating reciprocating flows. Using optimization techniques the constants of the \( v^2 - f \) model will be analyzed to determine which are the most sensitive to the reciprocating flow, which will hopefully lead to the ability to redefine the constants or establish a functional relationship relating to the reciprocating flow.

OpenFOAM is only available with a certain amount of turbulence models, which have been validated and verified, but it is believed that there are some more appropriate turbulence models which could be implemented in OpenFOAM and verified against the DNS reciprocating data. Since we have seen the apparent failure of the Reynolds’ analogy, con-
continued research will look into a new model which is an extension of the $v^2 - f$ model, which not only transports momentum fluctuations scales, but also thermal fluctuation scales (21) or (8). It is of the interest of the continued research to implement this model in OpenFOAM and test it in the reciprocating channel case. If the hypothesis that the thermal scales are incorrectly represented then the model should show improvement in the near wall heat transfer predictions.

Looking into the use of a modified eddy viscosity equation that can more accurately approximate the Reynolds’ stress would also be of use for continued research, one such model by Klein et. al. (22) uses a two timescale approach and an eddy viscosity definition that scales with the variation in mean shear. For the interest of keeping with the use of wall models to keep computations low, Popovac et. al. (27) has developed a compound wall treatment for use in non-equilibrium flows and has been specifically compared to the LES pulsatile channel results of Scotti (33), but more validation should be performed. In the end it is the hope that continued research will result in improved internal combustion engine simulations, which will help increase overall engine efficiency, decrease pollutant emissions, and provide overall better engine design practices.
Appendix A

A.1 Turbulence Models

A.1.1 $k - \varepsilon$

\[
\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \right) \frac{\partial k}{\partial x_j} \right] + \nu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} - \varepsilon. \tag{A.1}
\]

\[
\frac{\partial \varepsilon}{\partial t} + u_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \varepsilon}{\partial x_j} \right) + C_{\varepsilon 1} \varepsilon \nu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} - C_{\varepsilon 2} \frac{\varepsilon^2}{k}. \tag{A.2}
\]

\[
\nu_T = C_\mu \frac{k^2}{\varepsilon} \tag{A.3}
\]

\[
C_{\varepsilon 1} = 1.44 \quad C_{\varepsilon 2} = 1.92 \quad \sigma_\varepsilon = 1.3 \quad \sigma_k = 1.0 \tag{A.4}
\]

A.1.2 $k - \omega$

\[
\frac{\partial k}{\partial t} + u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \right) \frac{\partial k}{\partial x_j} \right] + \nu_T \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \frac{\partial \bar{u}_i}{\partial x_j} - \beta^* k \omega. \tag{A.5}
\]
\[
\frac{\partial \omega}{\partial t} + \bar{u}_j \frac{\partial \omega}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \right) \frac{\partial \omega}{\partial x_j} \right] + \alpha \frac{\omega}{k} \left[ \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} \right] - \beta \omega^2 \quad \text{(A.6)}
\]

\[
\nu_T = \frac{k}{\omega} \quad \text{(A.7)}
\]

\[
\alpha = 0.55 \quad \beta = 0.075 \quad \beta^* = 0.09 \quad \sigma = 0.5 \quad \sigma^* = 0.5 \quad \text{(A.8)}
\]

**A.1.3 \( k - \omega \) SST**

\[
\frac{\partial k}{\partial t} + \bar{u}_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \nu_T \right) \frac{\partial k}{\partial x_j} \right] + P_k - \beta^* k \omega \quad \text{(A.9)}
\]

\[
\frac{\partial \omega}{\partial t} + \bar{u}_j \frac{\partial \omega}{\partial x_j} = \alpha S^2 - \beta \omega^2 + \frac{\partial}{\partial x_j} \left[ \left( \nu + \sigma \nu_T \right) \frac{\partial \omega}{\partial x_j} \right] + 2(1 - F_1) \sigma \omega \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i} \quad \text{(A.10)}
\]

\[
\frac{\nu_T}{\max(a_1 \omega, SF_2)} = \frac{a_1 k}{\max(a_1 \omega, SF_2)} \quad \text{(A.11)}
\]

\[
S = \frac{1}{2} \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \quad \text{(A.12)}
\]

\[
F_1 = \tanh \left[ \min \left( \max \left( \frac{\sqrt{k}}{\beta^* \omega y}, \frac{500 \nu}{y^2 \omega} \right), \frac{4 \sigma k}{CD_k \omega y^2} \right) \right]^4 \quad \text{(A.13)}
\]

\[
F_2 = \tanh \left[ \max \left( \frac{\sqrt{k}}{\beta^* \omega y}, \frac{500 \nu}{y^2 \omega} \right) \right]^2 \quad \text{(A.14)}
\]
\[ P_k = \min \left( \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right), 10\beta^* k\omega \right) \]  
(A.15)

\[ CD_{k\omega} = \max \left( 2\sigma_{\omega 2} \frac{1}{\omega} \frac{\partial k}{\partial x_i} \frac{\partial \omega}{\partial x_i}, 10e^{-10} \right) \]  
(A.16)

\[ \phi = \phi_1 F_1 + \phi_2 (1 - F_1) \]  
(A.17)

\[ \alpha_1 = 0.55 \quad \alpha_2 = 0.44 \quad \beta_1 = 0.075 \quad \beta_2 = 0.828 \quad \beta^* = 0.09 \]
\[ \sigma_{k1} = 0.85 \quad \sigma_{k2} = 1.0 \quad \sigma_{\omega 1} = 0.5 \quad \sigma_{\omega 2} = 0.856 \]  
(A.18)

### A.1.4 \( k - \varepsilon \) LAUNDER-SHARMA

\[ \frac{\partial k}{\partial t} + \pi_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} - (\bar{\varepsilon} + D). \]  
(A.19)

\[ \frac{\partial \bar{\varepsilon}}{\partial t} + \pi_j \frac{\partial \bar{\varepsilon}}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu_T \frac{\partial \bar{\varepsilon}}{\sigma_\varepsilon \partial x_j} \right) + C_{\varepsilon 1} \bar{\varepsilon} \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} - C_{\varepsilon 2} f_2 \bar{\varepsilon}^2 + E. \]  
(A.20)

\[ \nu_T = C_\mu f_\mu \frac{k^2}{\bar{\varepsilon}} \]  
(A.21)

\[ \varepsilon = \bar{\varepsilon} + D \quad D = 2\nu \left( \frac{\partial \sqrt{k}}{\partial n} \right)^2 \]  
(A.22)

\[ E = 2\nu \nu_T \left( \frac{\partial^2 \pi}{\partial y^2} \right)^2, \]  
(A.23)
\[ D = 2\nu \left( \frac{\partial \sqrt{k}}{\partial n} \right)^2 \approx 2\nu \left( \nabla \sqrt{k} \right)^2. \]  
(A.24)

\[ f_\mu = \exp \left( \frac{-3.4}{(1 + Re_t/50)^2} \right) \]  
(A.25)

\[ f_2 = 1 - 0.3 \exp(-Re_t^2) \]  
(A.26)

\[ Re_T = \frac{k^2}{\nu \varepsilon} \]  
(A.27)

\[ C_\mu = 0.09 \quad \sigma_k = 1.0 \quad \sigma_\varepsilon = 1.3 \quad C_{\varepsilon 1} = 1.44 \quad C_{\varepsilon 2} = 1.92 \quad f_1 = 1.0 \]  
(A.28)

A.1.5 \( v^2 - f \)

\[
\frac{\partial k}{\partial t} + \pi_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_T}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} - \varepsilon. \]  
(A.29)

\[
\frac{\partial \varepsilon}{\partial t} + \pi_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\nu_T \varepsilon}{\sigma_\varepsilon \partial x_j} \right) + C_{\varepsilon 1} \frac{\varepsilon}{k} \nu_T \left( \frac{\partial \pi_i}{\partial x_j} + \frac{\partial \pi_j}{\partial x_i} \right) \frac{\partial \pi_i}{\partial x_j} - C_{\varepsilon 2} \frac{\varepsilon^2}{k}. \]  
(A.30)

\[
\frac{\partial v'^2}{\partial t} + \pi_j \frac{\partial v'^2}{\partial x_j} = kf - \frac{v'^2}{k} \varepsilon + \frac{\partial}{\partial x_j} \left[ \left( \nu + \frac{\nu_T}{\sigma_{v'^2}} \right) \frac{\partial v'^2}{\partial x_j} \right] \]  
(A.31)

\[
L^2 \nabla^2 f - f = \frac{C_1 - 1}{T} \left( \frac{v'^2}{k} - \frac{2}{3} \right) - C_2 \frac{P_k}{\varepsilon} \]  
(A.32)
\[ \nu_T = C_\mu \overline{v^2} T \]  
(A.33)

\[ L = C_L \max \left[ \frac{k^{3/2}}{\bar{\epsilon}}, C_\eta \left( \frac{\nu^3}{\bar{\epsilon}} \right)^{1/4} \right] \]  
(A.34)

\[ T = \max \left[ \frac{k}{\bar{\epsilon}}, C_T \left( \frac{\nu^3}{\bar{\epsilon}} \right)^{1/2} \right] \]  
(A.35)

\[ C_\mu = 0.22, \quad C_1 = 1.4, \quad C_2 = 0.3, \quad C_T = 6.0, \quad C_L = 0.23, \quad C_\eta = 70.0 \]  
(A.36)

### A.2 Example OpenFOAM Case Files

#### A.2.1 0/U

```c++
// * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

FoamFile
{
	only
  version 2.0;
  format ascii;
  class volVectorField;
  location "0";
  object U;
}

// * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

dimensions [0 1 -1 0 0 0 0];

internalField uniform (0 0 0);

boundaryField
{

```

115
bottomWall
{
    type fixedValue;
    value uniform (0 0 0);
}
frontAndBack
{
    type empty;
}
topWall
{
    type fixedValue;
    value uniform (0 0 0);
}
inout_half0
{
    type cyclic;
}
inout_half1
{
    type cyclic;
}

// ************************************************************************* //

A.2.2 \(0/\epsilon\)

FoamFile
{
    version 2.0;
    format ascii;
    class volScalarField;
    location "0";
    object epsilon;
}

dimensions [0 2 -3 0 0 0 0];
internalField uniform 0.0035;
boundaryField
{
    bottomWall
    {
        type zeroGradient;
    }
    frontAndBack
    {
        type empty;
    }
    topWall
    {
        type zeroGradient;
    }
    inout_half0
    {
        type cyclic;
    }
    inout_half1
    {
        type cyclic;
    }
}

// ************************************************************************* //

A.2.3 0/k

FoamFile
{
    version 2.0;
    format ascii;
    class volScalarField;
    location "0";
    object k;
}

// * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * //
dimensions [0 2 -2 0 0 0];

internalField uniform 0.031;

boundaryField
{
    bottomWall
    {
        type fixedValue;
        value uniform 1e-10;
    }
    frontAndBack
    {
        type empty;
    }
    topWall
    {
        type fixedValue;
        value uniform 1e-10;
    }
    inout_half0
    {
        type cyclic;
    }
    inout_half1
    {
        type cyclic;
    }
}

/*---------------------------------------------------------------------------*/
FoamFile
{
    version 2.0;
    format ascii;
    class volScalarField;
    location "0";
    object f;

A.2.4 0/F

$/)
A.2.5 0/NUT

FoamFile
{
    version 2.0;
    format ascii;
    ...


A.2.6 0/v2

```c++
class volScalarField;
location "0";
object nut;

// *************************************************************************/

dimensions [0 2 -1 0 0 0 0];
internalField uniform 1e-10;
boundaryField
{
    bottomWall
    {
        type calculated;
    }
    frontAndBack
    {
        type empty;
    }
    topWall
    {
        type calculated;
    }
    inout_half0
    {
        type cyclic;
    }
    inout_half1
    {
        type cyclic;
    }
}

// *************************************************************************/
```
{ 
  version 2.0;
  format ascii;
  class volScalarField;
  location "0";
  object v2;
}

// ************************************************************************* //

dimensions [0 2 -2 0 0 0 0];
internalField uniform 0.02067;

boundaryField
{
  bottomWall
  {
    type fixedValue;
    value uniform 1e-10;
  }
  frontAndBack
  {
    type empty;
  }
  topWall
  {
    type fixedValue;
    value uniform 1e-10;
  }
  inout_half0
  {
    type cyclic;
  }
  inout_half1
  {
    type cyclic;
  }
}

// ************************************************************************* //
\verbatim
FoamFile

{
    version 2.0;
    format ascii;
    class volScalarField;
    location "0";
    object p;
}

// ************************************************************************* //
dimensions [0 2 -2 0 0 0 0];

internalField uniform 0;

boundaryField
{
    bottomWall
    {
        type zeroGradient;
    }

    frontAndBack
    {
        type empty;
    }

    topWall
    {
        type zeroGradient;
    }

    inout_half0
    {
        type cyclic;
    }

    inout_half1
    {
        type cyclic;
    }
}

// ************************************************************************* //

A.2.8 0/T
A.2.9 CONSTANT/RASPROPERTIES
A.2.10  CONSTANT/transportProperties

define the pressure gradient value

dpdx  dpdx  [ 0 1 -2 0 0 0 0 ] ( 1 0 0 );
//Freq = 2pi/Period T40Freq = 0.157
Freq  Freq  [ 0 0 0 0 0 0 ] 0.20943951;
//Constants for Temperature

// Reference temperature
TRef TRef [0 0 0 1 0 0 0] 0;

// Laminar Prandtl number
Pr Pr [0 0 0 0 0 0 0] 0.7;

// Turbulent Prandtl number
Prt Prt [0 0 0 0 0 0 0] 0.9;

transportModel Newtonian;

//nu = 1/Re
nu nu [0 2 -1 0 0 0 0] 0.0005;

rho rho [1 -3 0 0 0 0 0] 1;

CrossPowerLawCoeffs
{
    nu0 nu0 [0 2 -1 0 0 0 0] 1e-06;
    nuInf nuInf [0 2 -1 0 0 0 0] 1e-06;
    m m [0 0 1 0 0 0 0] 1;
    n n [0 0 0 0 0 0 0] 1;
}

BirdCarreauCoeffs
{
    nu0 nu0 [0 2 -1 0 0 0 0] 1e-06;
    nuInf nuInf [0 2 -1 0 0 0 0] 1e-06;
    k k [0 0 1 0 0 0 0] 0;
    n n [0 0 0 0 0 0 0] 1;
}

// ************************************************************************* //

A.2.11 CONSTANT/TURBULENCEPROPERTIES

// ************************************************************************* //
A.2.12  SYSTEM/CONTROLDict

```cpp
#include<system/controlDict.I>

FoamFile
{
    version 2.0;
    format ascii;
    class dictionary;
    location "system";
    object controlDict;
}

// ************************************************************************* //
application pimpleFoamPulsTempNew;
startFrom latestTime;
startTime 0;
stopAt endTime;
endTime 690;
maxCo 1.0; // Or other Courant number you wish
adjustTimeStep no; // Or no
deltaT 0.0003;
writeControl timeStep;
// ************************************************************************* //
writeInterval 3125;
purgeWrite 0;
writeFormat ascii;
writePrecision 8;
writeCompression off;
timeFormat general;
timePrecision 6;
runTimeModifiable true;
functions {
}

A.2.13 SYSTEM/fvSchemes
default Gauss linear;

grad(p) Gauss linear;

grad(U) Gauss linear;

}

divSchemes

{
  default none;
  div(phi,U) Gauss limitedLinearV 1;
  div(phi,k) Gauss limitedLinear 1;
  div(phi,epsilon) Gauss limitedLinear 1;
  div(phi,R) Gauss limitedLinear 1;
  div(R) Gauss linear;
  div(phi,nuTilda) Gauss limitedLinear 1;
  div((nuEff*dev(T(grad(U))))) Gauss linear;
  div(phi,T) Gauss limitedLinear 1;
  div(phi,v2) Gauss limitedLinear 1;

}

laplacianSchemes

{
  default none;
  laplacian(nuEff,U) Gauss linear corrected;
  laplacian(rAUf,p) Gauss linear corrected;
  laplacian(DkEff,k) Gauss linear corrected;
  laplacian(DepsilonEff,epsilon) Gauss linear corrected;
  laplacian(DREff,R) Gauss linear corrected;
  laplacian(DnuTildaEff,nuTilda) Gauss linear corrected;
  laplacian(alphaEff,T) Gauss linear corrected;
  laplacian(f) Gauss linear corrected;
  laplacian(DkEff,v2) Gauss linear corrected;
  laplacian(alphaEff,T) Gauss linear corrected;

}

interpolationSchemes

{
  default linear;
  interpolate(U) linear;

}

snGradSchemes

{
  default corrected;

}

fluxRequired

{
  default no;
  p ;

}
A.2.14 SYSTEM/fvSolution

FoamFile
{
    version 2.0;
    format ascii;
    class dictionary;
    location "system";
    object fvSolution;
}

// ************************************************************************* //

solvers
{
    p
    {
        solver GAMG;
        tolerance 1e-7;
        relTol 0.01;

        smoother DICGaussSeidel;

        cacheAgglomeration true;
        nCellsInCoarsestLevel 10;
        agglomerator faceAreaPair;
        mergeLevels 1;
    }

    pFinal
    {
        $p;
        relTol 0;
    }

    "(U|k|epsilon|T|f|v2)"
    {
        solver smoothSolver;
        smoother symGaussSeidel;
        tolerance 1e-07;
    }

    // ************************************************************************* //
A.3 Finite Difference Schemes

Derive finite difference scheme for first derivative of function $g$:
for $j = 0$

\[
\frac{dg}{dz} = dg_j + eg_{j+1} + fg_{j+2}
\]

\[
g_{j+1} = g_j + (z_{j+1} - z_j) \frac{dg}{dz} + \frac{(z_{j+1} - z_j)^2}{2} \frac{d^2 g}{dz^2}
\]

\[
g_{j+2} = g_j + (z_{j+2} - z_j) \frac{dg}{dz} + \frac{(z_{j+2} - z_j)^2}{2} \frac{d^2 g}{dz^2}
\]

\[g : \quad d + e + f = 0\]

\[
\frac{dg}{dz} : \quad e(z_{j+1} - z_j) + f(z_{j+2} - z_j) = 1
\]

\[
\frac{d^2 g}{dz^2} : \quad \frac{e(z_{j+1} - z_j)^2}{2} + \frac{f(z_{j+2} - z_j)^2}{2} = 0
\]

Now Solve for $d,e,f$

\[d = -e - f\]

\[e = -f \frac{(z_{j+2} - z_j)^2}{(z_{j+1} - z_j)^2}\]

\[f = \frac{-1}{\frac{(z_{j+2} - z_j)^2}{(z_{j+1} - z_j)} - (z_{j+2} - z_j)}\]
for $j = 1 \rightarrow (Nz - 1)$

$$\frac{dg}{dz} = dg_j + eg_{j-1} + fg_{j+1}$$

$$g_{j-1} = g_j + (z_{j-1} - z_j) \frac{dg}{dz} + \frac{(z_{j-1} - z_j)^2}{2} \frac{d^2 g}{dz^2}$$

$$g_{j+1} = g_j + (z_{j+1} - z_j) \frac{dg}{dz} + \frac{(z_{j+1} - z_j)^2}{2} \frac{d^2 g}{dz^2}$$

\[ g : \quad d + e + f = 0 \]

\[ \frac{dg}{dz} : \quad e(z_{j-1} - z_j) + f(z_{j+1} - z_j) = 1 \]

\[ \frac{d^2 g}{dz^2} : \quad \frac{e(z_{j-1} - z_j)^2}{2} + \frac{f(z_{j+1} - z_j)^2}{2} = 0 \]

Now Solve for $d, e, f$

$$d = -e - f$$

$$e = -f \frac{(z_{j+1} - z_j)^2}{(z_{j-1} - z_j)^2}$$

$$f = \frac{-1}{\frac{(z_{j+1} - z_j)^2}{(z_{j-1} - z_j)} - (z_{j+1} - z_j)}$$
for $j = Nz$

$$\frac{dg}{dz} = dg_j + eg_{j-1} + fg_{j-2}$$

$$g_{j-1} = g_j + (z_{j-1} - z_j) \frac{dg_j}{dz} + \frac{(z_{j-1} - z_j)^2}{2} \frac{d^2 g}{dz^2}$$

$$g_{j-2} = g_j + (z_{j-2} - z_j) \frac{dg_j}{dz} + \frac{(z_{j-2} - z_j)^2}{2} \frac{d^2 g}{dz^2}$$

$$g : \quad d + e + f = 0$$

$$\frac{dg}{dz} : \quad e(z_{j-1} - z_j) + f(z_{j-2} - z_j) = 1$$

$$\frac{d^2 g}{dz^2} : \quad \frac{e(z_{j-1} - z_j)^2}{2} + \frac{f(z_{j-2} - z_j)^2}{2} = 0$$

Now Solve for $d,e,f$

$$d = -e - f$$

$$e = -f \frac{(z_{j-2} - z_j)^2}{(z_{j-1} - z_j)^2}$$

$$f = \frac{-(z_{j-2} - z_j)^2}{(z_{j-1} - z_j)^2} - (z_{j-2} - z_j)$$

### A.4 Analytical Solution Laminar reciprocating Flow

\[
\frac{\partial \pi}{\partial t} = A \cos(\omega t) + \nu \frac{\partial^2 \pi}{\partial z^2} \quad (A.37)
\]

Rewrite the cos in complex form.

\[
\frac{\partial \hat{u}}{\partial t} = A e^{i\omega t} + \nu \frac{\partial^2 \hat{u}}{\partial z^2} \quad (A.38)
\]

Define some terms
\[ \tau = \omega t \quad \hat{u} = \frac{\hat{v}}{\omega} A \quad \eta = 1 - \frac{z}{h} \quad (A.39) \]

Substitute in the terms

\[ \frac{A}{\omega} \frac{\partial \hat{v}}{\partial \tau} = Ae^{i\tau} + \frac{\nu A}{h^2 \omega} \frac{\partial^2 \hat{v}}{\partial \eta^2} \quad (A.40) \]

\[ \frac{\partial \hat{v}}{\partial \tau} = e^{i\tau} + \frac{\nu}{h^2 \omega} \frac{\partial^2 \hat{v}}{\partial \eta^2} \quad (A.41) \]

Define a few more terms

\[ \frac{1}{2\Lambda^2} = \frac{\nu}{h^2 \omega} \quad \Lambda = \frac{h}{\sqrt{2\nu/\omega}} \quad (A.42) \]

Substitute in \( \Lambda \)

\[ \frac{\partial \hat{v}}{\partial \tau} = e^{i\tau} + \frac{1}{2\Lambda^2} \frac{\partial^2 \hat{v}}{\partial \eta^2} \quad (A.43) \]

\[ \hat{v}(\eta, \tau) = e^{i\tau} \hat{f}(\eta) \rightarrow \hat{f} = 1 + \frac{1}{2\Lambda^2} \hat{f}'' \quad (A.44) \]

\[ \hat{f}'' = i2\Lambda^2 \left( \hat{f} + \hat{i} \right) \quad \hat{g} = \hat{f} + \hat{i} \quad (A.45) \]

\[ \hat{g}'' = i2\Lambda^2 \hat{g} \quad (A.46) \]

\[ \eta = \pm 1 \quad \hat{f} = 0 \rightarrow \hat{g}_{\eta=\pm 1} = \hat{i} \quad (A.47) \]
\[ \hat{g} = i \frac{\cosh(\sqrt{2i \Lambda \eta})}{\cosh(\sqrt{2i \Lambda})} \]  
(A.48)

\[ \hat{v} = i \left( \frac{\cosh(\sqrt{2i \Lambda \eta})}{\cosh(\sqrt{2i \Lambda})} - 1 \right) e^{i \gamma} \]  
(A.49)

\[ \sqrt{2i} = \pm (1 + i) \]  
(A.50)

\[ v = Re(\hat{v}) \quad u = \frac{A}{\omega} Re(\hat{v}) \]  
(A.51)

\[ \hat{v} = i e^{i \gamma} \left( \frac{\cosh(- (1 + i) \Lambda \eta)}{\cosh(- (1 + i) \Lambda)} - 1 \right) \]  
(A.52)

\[ \frac{d\hat{v}}{d\eta} = (-1 + i) \Lambda e^{i \gamma} \left( \frac{\sinh((1 + i) \Lambda \eta)}{\cosh((1 + i) \Lambda)} \right) \]  
(A.53)

\[ \frac{d^2\hat{v}}{d\eta^2} = -2 \Lambda^2 e^{i \gamma} \left( \frac{\cosh((1 + i) \Lambda \eta)}{\cosh((1 + i) \Lambda)} \right) \]  
(A.54)

\[ \frac{d\hat{v}}{d\eta} = \frac{d\hat{v}}{d\eta} \frac{d\eta}{dz} = -\frac{1}{h} \frac{d\hat{v}}{d\eta} \]  
(A.55)
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