Assessing the Roof Structure of the Breeding Barn Using Truss Member Resonant Frequencies

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ASSESSING THE ROOF STRUCTURE OF THE BREEDING BARN USING TRUSS MEMBER RESONANT FREQUENCIES

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ABSTRACT

The motivation for this research was to apply methods of vibrations testing in order to determine axial loads in the pin-ended truss members of the Breeding Barn. This method of vibrations testing was necessary in order to determine the in-situ axial loads of the truss members in the barn. Other common methods, such as strain gauges, were not useful for this application. This is because strain gauges can only detect changes in strain and therefore only changes in load. However due to the size and weight of the roof at the Breeding Barn, significant axial loads are produced in the truss members. This in-situ axial load due to the dead load of the roof is a significant portion of any additional loading and cannot be ignored. The ultimate goal of determining the axial loads in the truss members was to develop a model for the roof structure of the barn that accurately predicts axial loads in the truss members over a range of loading conditions. Developing such a model was important in order to make a structural assessment of the Breeding Barn’s roof structure.

In order to determine the axial loads in the truss members, acceleration time histories of the individual truss members were collected using wireless accelerometers provided by MicroStrain of Williston, Vermont. Using the Fourier transform, power spectral densities were produced from the raw acceleration time histories. It was from these plots that the resonant frequencies of the truss members were determined. Knowing the resonant frequencies for a member and the beam vibration equation developed for pin-ended members, the axial load of the truss member were calculated. This process was done for each wrought iron truss member for three separate loading conditions. The purpose of this was to provide enough experimental data so that it could be compared with predictions of several proposed frame models of the barn’s roof structure. Ultimately a model was chosen that best predicted the axial loads in the truss members based upon the three loading combinations tested. Using this frame model, an assessment of the barn’s roof structure could be made.
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CHAPTER 1: INTRODUCTION

1.1. Motivation

The motivation for this research was to apply methods of vibrations testing in order to determine axial loads in the pin-ended truss members of the Breeding Barn. This method of vibrations testing was necessary in order to determine the in-situ axial loads of the truss members in the barn. Other common methods, such as strain gauges, were not useful for this application. This is because unless the reference state is the unstressed state of the member, strain gauges will not measure existing axial load but instead the change in load from the reference state by means of the measured strain. However due to the size and weight of the roof at the Breeding Barn, significant axial loads are produced in the truss members. This in-situ axial load due to the dead load of the roof is a significant portion of any additional loading and cannot be ignored. The ultimate goal of determining the axial loads in the truss members was to develop a model for the roof structure of the barn that accurately predicts axial loads in the truss members over a range of loading conditions. Developing such a model was important in order to make a structural assessment of the Breeding Barn’s roof structure.

1.2 Background

1.2.1. History of Breeding Barn

The Breeding Barn is a large stately barn located at Shelburne Farms on the shores of Lake Champlain in Shelburne, Vermont. The barn was built between 1889 and 1891 for William Seward Webb. Robert Robertson designed the barn as well as many of the other structures located on the grounds. Robertson was a renowned architect, and is well known for being one of the architects of Grand Central Terminal in New York City. The barn was constructed using the best available materials and cost $134,000 to build, yet was only
appraised at $49,000. The wrought iron used in the principal trusses throughout the barn was provided by Post & McCord, the same company that supplied the steel used to construct the Empire State Building. A photo of the original materials list supplied by Post & McCord can be seen in Figure 1.1. The barn was originally built to house Mr. Webb’s Hackney horse breeding operation. It was Webb’s goal to introduce the Hackney horse, which he believed to be a well balanced work horse as well as carriage horse, to Vermont. However, people did not so easily buy into the Hackney horse. This coupled with Webb’s declining health and the birth of the automobile doomed this operation by 1904.

![Figure 1.1: Breeding Barn materials list supplied by Post & McCord.](image_url)
In 1913 the southern acres of Shelburne Farms, including the Breeding Barn, was deeded to J. Watson and Electra Havemeyer Webb. While no longer housing the Hackney horse breeding operation, it was commonly used for polo matches as well as fox hunts. Overtime the barn was used more and more infrequently and came into disrepair. The demolition of the barn was even considered, but due to a lack of funds did not occur. In 1980 Shelburne Farms, including the Breeding Barn, was placed on the National Register of Historic Place. The southern acres property was managed by J. Watson and Electra Havemeyer Webb and their children until 1986. In 1986 the southern acres of Shelburne Farms was transferred to the Shelburne Museum. In 1994, in an effort to preserve the property, the barn was returned to Shelburne Farms, and a major effort was made to stabilize and restore the barn. In 2001 Shelburne Farms was designated a National Historic Landmark.

1.2.2. Prior Work & Assessments

In 1990, the Shelburne Museum and Shelburne Farms were contemplating transferring the southern acres including the Breeding Barn back to Shelburne Farms. In order to determine the level of work and commitment which would be required to rehabilitate the barn if it were reacquired, a series of studies were commissioned by Shelburne Farms. These included a structural assessment conducted by Civil Engineering Associates (CEA) as well as the Breeding Barn Stabilization and Rehabilitation report prepared by Martin Tierney.

Emergency stabilization of the barn began in 1995 and continued for several years. Work completed during this stabilization included replacement and repair of many of the rotted timbers, foundation repairs, repair of exterior woodwork, and replacement of the roof. In addition to this, a fire detection system and a dry pipe suppression system was installed in 2001.
In 2003 a team was assembled to prepare a conservation plan for the entire Breeding Barn complex. This was funded partially by an Architectural Conservation Planning Grant from the Getty Foundation. This team included Smith-Alvarez-Sienkiewycz (architects), Mel Doherty (structural engineer), Patricia O'Donnell of Heritage Landscapes (landscape architect), Crothers Environmental Group (hazardous material analysis), and Mary Jo Llewellyn (finish analysis). A report was completed in 2004 by this team which suggested a conventional approach to strengthening the barn with the addition of steel reinforcement.

Due to the severe impact such an approach would have on the integrity and historical significance of the barn, a second team was formed to alternative to the structural stabilization of the barn. This team is composed of Douglas Porter (architectural conservator), David Fishchetti (structural engineer), Ron Anthony (wood scientist), Scott Kreilick (metallurgist), and Elizabeth Louden (documentation). The goals of this group are as follows:

(i) Accurate and painstaking examination of surviving fabric to discover the nature and condition of materials and connections

(ii) Characterization of timber and metal elements, using non-destructive and quasi non-destructive testing techniques to the fullest extent possible

(iii) Rational selection of design values based on the conditions survey, materials testing, and review of the original construction documents and original design methodologies

(iv) Reduction of factors of safety through exhaustive knowledge of the building

(v) Identification of overstresses through careful modeling and analysis

(vi) Development of a HABS-level documentation package, to be contributed to the Library of Congress upon completion of the project
1.2.3. Breeding Barn Principal Truss

The roof structure of the Breeding Barn consists of several elements including the side aisles of the barn, the x-bracing, and the principal truss itself. What instantly strikes someone upon walking into the barn, though, is the massive openness made possible by the barn's roof truss. A composite truss, constructed of wood and iron is used to support the roof throughout the 360' x 72' indoor space. The truss used in the Breeding Barn is derived from a composite truss designed by Camille Polonceau during the 1830's. This form of truss was widely used for moderate to long spans such as those found in train sheds during that time period.

Southern pine is used for the members in the top chord of the truss, while wrought iron is used throughout the remaining members. Several of the wrought iron members in the truss are actually composed of double members. These double members run in parallel with each other and work in tandem. The principal truss used in the Breeding Barn is detailed in Figure 1.2, while the member properties are summarized in Table 1.1. A numbering scheme was developed in order to easily identify the individual truss members. This numbering scheme of the nodes and elements of the principal truss can be seen in Figure 1.2 and continues throughout this paper. In the case of double members, the orientation to east and west was used to tell the difference between them.
Figure 1.2: Layout of principal truss used throughout the Breeding Barn

The principal truss spans the 72' width of the barn and is placed at every 24'. Purlins, measuring 8" x 9", span the principal trusses. Each of these purlins is reinforced with a king post truss constructed of wrought iron. A total of seven purlins come onto each principal truss to transfer loads from the roof. These purlins are located at nodes 2, 4, 5, 6, 7, 8, and 10 from Figure 1.2. Each principal truss rests 28' off the ground of the barn atop a pair of continuous 12" x 12" southern pine columns.
Table 1.1: Summary of material properties used in Breeding Barn principal truss

<table>
<thead>
<tr>
<th>Member #</th>
<th>Material</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Thru 10</td>
<td>Southern Pine</td>
<td>10&quot; x 12&quot;</td>
</tr>
<tr>
<td>11</td>
<td>Wrought Iron</td>
<td>(2) 1&quot; Dia. Rod</td>
</tr>
<tr>
<td>12</td>
<td>Wrought Iron</td>
<td>1.5&quot; Dia. Rod</td>
</tr>
<tr>
<td>13</td>
<td>Wrought Iron</td>
<td>(2) 1&quot; Dia. Rod</td>
</tr>
<tr>
<td>14</td>
<td>Wrought Iron</td>
<td>(4) 3&quot;x3&quot; x 3/8&quot; Angle</td>
</tr>
<tr>
<td>15</td>
<td>Wrought Iron</td>
<td>(2) 3/4&quot; Dia. Rod</td>
</tr>
<tr>
<td>16</td>
<td>Wrought Iron</td>
<td>(2) 3/4&quot; Dia. Rod</td>
</tr>
<tr>
<td>17</td>
<td>Wrought Iron</td>
<td>(4) 3&quot;x3&quot; x 3/8&quot; Angle</td>
</tr>
</tbody>
</table>

1.3. Project Summary

By applying methods of vibrations testing, the axial loads in the wrought iron truss members at the Breeding Barn can be determined. This is accomplished by measuring the acceleration time histories of vibrating truss members using wireless accelerometers. By applying the Fourier transform, a power spectral density can be produced from this raw data. It is from the power spectral density that the resonant frequencies of the truss members can be established. Using the characteristic or frequency equations developed further in this paper, the axial loads for the truss members can be calculated.

Axial loads of the wrought iron truss members will determined for three different load combinations. As the load combination increases, so will the axial load it produces in the truss members. The increased load applied to the truss will be accomplished using 1000 lbs.
concrete blocks. Several frame models have been developed to model the behavior of the roof structure at the Breeding Barn. Using the three load combinations, the axial loads predicted by the roof structure models will be compared to those actually measured at the barn. The goal of this comparison is to determine which model, if any, accurately predicts the axial loads in the truss members for a range of loading. Finally upon selecting a model, the axial loads for a full snow load will be predicted. A major concern at the Breeding Barn is that a full snow load could possibly cause some of the members in the truss to yield without further reinforcement. The axial loads predicted by the final model will be used to determine whether or not this is truly a concern.
CHAPTER 2: EXPERIMENT

2.1. Purpose

The primary motive for conducting this experiment was to determine the axial loads experienced in the truss members, specifically the wrought iron members, for a range of different loading scenarios. Of particular interest was confirming that the resonant frequencies of the members increased with additional loading as the load combinations were increased. Also important in regards to the condition of the structure were the in-situ axial loads. Determining the forces in the truss due to only the dead load was crucial in order to assess the structure as it stands on a daily basis. In addition, the axial loads determined from this experiment could then be compared to the axial loads predicted by different plane frame models of the roofing structure. From this comparison it could be determined which analysis best fit the actual conditions experienced in the truss. This was extremely useful when trying to determine the axial forces in the truss due to increasing loadings that could not safely be applied to the truss because the possibility of overstressing the truss members.

2.2. Test Set-Up

A loading placed on the roof would first be transmitted to the tongue and groove sheathing atop the common rafters. The tongue and groove sheathing, which is spread throughout the entire roof, would then transfer the load to the common rafters. From the common rafters, the load would be transferred to the purlins that span the principal trusses. The purlins would then transfer this load to the principal truss. There are three
purlins that run between the top chords on both the north and south side of the principal trusses as well as an additional purlin at the apex of the truss. Since it was not reasonable to place a load on the roof, a similar loading path was reproduced for the experiment by directly loading the purlins. Different loadings could be produced by simply varying the weight applied to each purlin as well as the position along the purlin where the weight was applied. A schematic detailing the materials used in the roofing structure of the Breeding Barn can be seen below in figure 2.1.

![Figure 2.1: Schematic of Breeding Barn's roof structure](image-url)
The first step in the experiment was choosing the primary truss on which the tests would be conducted. The primary concern in choosing the truss was making sure that the layout of the roof and the trusses to either side of it were identical. This ensured a symmetrical dead loading and also defined a simple load path from the purlins to the truss. For this reason the fourth primary truss (not including the double truss supporting the lantern) east of the main entrance of the barn was chosen. The location of this truss avoided the more complicated roof details of the lantern located at the entrance and the large dormers located at the far ends of the barn. A total of 14 concrete blocks, each weighing 1000 lb, were used to load the truss. The concrete blocks were chosen as an inexpensive method for loading the truss for the experiment. Other techniques were considered, such as loading the truss with large containers of water, but these were much more expensive in comparison. A weight of 1000 lb for the concrete blocks was chosen because it made the blocks a manageable size and allowed them to still be lifted manually. It was also determined prior to the experiment through a frame analysis that loading the truss with the 1000 lb concrete blocks would cause measurable increases in the axial loads in the truss members.

To load the truss, the 14 concrete blocks needed to be suspended from the purlins. In order to do this, structural rigging straps were secured to each purlin east and west of the primary truss being tested. In total, 14 separate purlins were prepared to receive the concrete blocks. Static climbing rope was then used to join the rigging straps attached to the purlins to the concrete blocks at the ground. A photo of this setup can be seen in figure 2.2.
The concrete blocks were lifted by means of come-alongs and chain hoists. This was done by first attaching one end of the come-along to a loop knotted into the static climbing rope hanging from the purlin. The other end was then hooked to an additional rigging strap tied to the concrete block. With this in place the come-along could be used to lift the concrete block above the ground. The configuration of the concrete block and come-along can be seen in figure 2.3. Although static climbing rope rated for roughly 5700 lb was used, there was a large amount of stretch in the rope from lifting the 1000 lb concrete blocks. Due to this stretching, the come-alongs were not able to lift the blocks much more than a few inches above the ground. This was not a problem though since the full load of the concrete block was transferred to the truss.
regardless of how high it was raised. The stretching of the rope did mean however that the concrete blocks needed to be lifted periodically in order to avoid having them come back to rest on the ground. Another consideration when lifting the concrete blocks was to avoid creating an unbalanced or unsymmetrical loading on the truss. This was done by lifting blocks that were located at opposite ends and sides of each other. For example if a block at the far northeast purlin of the truss was lifted, a second block at the far southwest purlin was also lifted.

Figure 2.3: Example of connection between come-along and concrete block
2.3. Test Load Combinations

A total of three different load combinations were examined during testing at the Breeding Barn. The first load combination considered was simply the dead weight of the roofing system. This loading scenario was one of the most important to consider in order to evaluate the safety of the barn on a day-to-day basis. For example, if the axial loads in the members for this loading were determined to be too high, the closing of the structure to the public would need to be considered.

Two additional loading combinations were also considered for the testing at the barn. Load combination 1 involved applying an additional load, which is described in detail further in this chapter, to the dead weight that the truss was already experiencing. The goal of this additional loading was to confirm that the resonant frequencies, and thus the axial loads in the members increased. The original idea for load combination 2 was to double the applied load from load combination 1. The purpose of this set-up was to see if the increase in frequencies from load combination 1 to load combination 2 doubled as expected. However, the increase in frequencies for load combination 1 was not as large as predicted. For this reason the additional applied loading was not simply double for load combination 2. Instead a loading scenario which would cause higher axial loads in the truss members was used.
2.3.1. Load Combination 0 – Dead Load

The roofing structure at the Breeding Barn makes up a significant weight which needs to be considered when determining the in-situ axial loads of the truss members. The dead weight of the roof was estimated to help accurately predict these loads. The principal trusses at the Breeding Barn are spaced every 24 ft. The weight of the roof structure including the principal truss was determined for this 24 ft. span. The materials used throughout the roof structure are primarily southern pine, wrought iron, and copper. The southern pine is used for all of the wooden members in the roofing system including the top chord of the truss, purlins, common rafters, and roof tongue and groove sheathing. The wrought iron makes up the remaining members of the truss, while copper is used on the roof. The properties used for these materials in calculating the weight of the roof are detailed below in table 2.1.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Southern Pine</td>
<td>.02 $\text{lb/}\text{in}^3$</td>
</tr>
<tr>
<td>Wrought Iron</td>
<td>.264 $\text{lb/}\text{in}^3$</td>
</tr>
<tr>
<td>Copper Roof</td>
<td>1 $\text{lb/ft}^2$</td>
</tr>
</tbody>
</table>

Table 2.1: Roofing structure material properties
The designs of the roofing structure as well as the dimensions of most of the lumber used in it are known fairly well from Robertson’s original designs. One exception to this was the thickness of the tongue and groove sheathing covering the roof. For the purpose of this calculation a thickness of \( \frac{1}{2} \) in was used for the sheathing. This is a reasonable value, but it’s possible that the thickness of the sheathing could be as much as 1 in. Knowing the roof structure design, an estimation of the weight of the roof was determined. Table 2.2 outlines the estimation of the roof structures weight.

**Table 2.2: Materials breakdown for estimation of roof structure weight**

<table>
<thead>
<tr>
<th>Description</th>
<th>Quantity</th>
<th>Length</th>
<th>Area (( \text{in}^2 ))</th>
<th>Density ( \text{lb} \text{in}^3 )</th>
<th>Weight (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Chord</td>
<td>1</td>
<td>85.62</td>
<td>120</td>
<td>0.02</td>
<td>2492</td>
</tr>
<tr>
<td>Members 11, 13</td>
<td>2</td>
<td>22.41</td>
<td>1.57</td>
<td>0.264</td>
<td>226</td>
</tr>
<tr>
<td>Member 12</td>
<td>1</td>
<td>27.50</td>
<td>1.77</td>
<td>0.264</td>
<td>154</td>
</tr>
<tr>
<td>Members 14, 17</td>
<td>2</td>
<td>10.0</td>
<td>8.44</td>
<td>0.264</td>
<td>537</td>
</tr>
<tr>
<td>Members 15, 16</td>
<td>2</td>
<td>24.62</td>
<td>0.88</td>
<td>0.264</td>
<td>138</td>
</tr>
<tr>
<td>Purlins</td>
<td>7</td>
<td>24</td>
<td>72</td>
<td>0.02</td>
<td>2903</td>
</tr>
<tr>
<td>Common Rafters</td>
<td>48</td>
<td>43</td>
<td>18</td>
<td>0.02</td>
<td>8916</td>
</tr>
<tr>
<td>Sheathing</td>
<td>1</td>
<td>( \frac{1}{2} ) in thick</td>
<td>2064 ft(^2)</td>
<td>0.02</td>
<td>2972</td>
</tr>
<tr>
<td>Copper Roof</td>
<td>1</td>
<td>-</td>
<td>2064 ft(^2)</td>
<td>( \frac{1}{\text{ft}^2} )</td>
<td>2064</td>
</tr>
</tbody>
</table>

\[ \Sigma \text{Weight} = 20402 \]
For a 24 ft. span, the estimated weight of the roof structure is 20402 lbs. This weight acts over a projected area of 72 ft. x 24 ft. or 1728 ft.². The dead weight of the roof can simply be calculated as distributed load of \( \frac{20402 \text{ lb}}{1728 \text{ ft}^2} \) or 11.81 lb/ft². Using this as the dead load of the roof and knowing the horizontal spacing of the purlins, the dead load of the roof can be calculated and applied to the purlins as point loads. A description of this loading is shown below in table 2.3.

<table>
<thead>
<tr>
<th>Node Description</th>
<th>Node Number</th>
<th>Dead Load (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Purlin 1</td>
<td>2</td>
<td>2.53</td>
</tr>
<tr>
<td>Northern Purlin 2</td>
<td>4</td>
<td>2.53</td>
</tr>
<tr>
<td>Northern Purlin 3</td>
<td>5</td>
<td>2.53</td>
</tr>
<tr>
<td>Apex Purlin</td>
<td>6</td>
<td>2.53</td>
</tr>
<tr>
<td>Southern Purlin 3</td>
<td>7</td>
<td>2.53</td>
</tr>
<tr>
<td>Southern Purlin 2</td>
<td>8</td>
<td>2.53</td>
</tr>
<tr>
<td>Southern Purlin 1</td>
<td>10</td>
<td>2.53</td>
</tr>
</tbody>
</table>
2.3.2. Load Combination 1 – Dead Load & Load Test 1

Load combination 1 included the applied load from the first loading test in addition to the dead load of the roof. The first load test involved applying 1000 lbs at each node along the top chord of the truss. This was accomplished by hanging a concrete block mid-span along each purlin on both sides of the truss being tested. Since each concrete block weighed 1000 lbs and the block was located mid-span along the purlin, half of its load was transferred to each truss connected to the purlin. Therefore 1000 lbs was applied at each node where the purlins came into the top chord of the truss. The result of this loading was that each node on the truss was loaded with 2.53 k from the dead load of the roof plus an additional 1000 lbs for load test 1 for a combined loading of 3.53 k at each node. This loading is summarized below in table 2.4.

Table 2.4: Detailed loading of principal truss under load combination # 1

<table>
<thead>
<tr>
<th>Node Description</th>
<th>Node Number</th>
<th>Applied Load (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Purlin 1</td>
<td>2</td>
<td>1.0</td>
</tr>
<tr>
<td>Northern Purlin 2</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>Northern Purlin 3</td>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>Apex Purlin</td>
<td>6</td>
<td>1.0</td>
</tr>
<tr>
<td>Southern Purlin 3</td>
<td>7</td>
<td>1.0</td>
</tr>
<tr>
<td>Southern Purlin 2</td>
<td>8</td>
<td>1.0</td>
</tr>
<tr>
<td>Southern Purlin 1</td>
<td>10</td>
<td>1.0</td>
</tr>
</tbody>
</table>
2.3.3 Load Combination 2 – Dead Load & Load Test 2

Load combination 2 included the applied load from the second loading test in addition to the dead load of the roof. The second loading consisted of loading nodes 5, 6, and 7 with an applied load of 3000 lbs. Nodes 4 and 8 were loaded with an additional 1000 lb. The remaining nodes were loaded with any additional weight. This was accomplished by loading the ridgeline and the two purlins below it with two blocks directly at the truss being tested and an additional block at mid-span of the purlin on either side. A concrete block was also hung directly next to the truss from the next purlin down the roof on either side. The result of this loading was that nodes 2 and 10 only received the dead loading of 2.53 k. Nodes 4 and 8 were loaded with a combined 3.53 k, while nodes 5, 6, and 7 were loaded with a combined 6.53 k. This load combination is summarized below in table 2.5.

Table 2.5: Detailed loading of principal truss under load combination # 2

<table>
<thead>
<tr>
<th>Node Description</th>
<th>Node Number</th>
<th>Applied Load (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northern Purlin 1</td>
<td>2</td>
<td>0.0</td>
</tr>
<tr>
<td>Northern Purlin 2</td>
<td>4</td>
<td>1.0</td>
</tr>
<tr>
<td>Northern Purlin 3</td>
<td>5</td>
<td>3.0</td>
</tr>
<tr>
<td>Apex Purlin</td>
<td>6</td>
<td>3.0</td>
</tr>
<tr>
<td>Southern Purlin 3</td>
<td>7</td>
<td>3.0</td>
</tr>
<tr>
<td>Southern Purlin 2</td>
<td>8</td>
<td>1.0</td>
</tr>
<tr>
<td>Southern Purlin 1</td>
<td>10</td>
<td>0.0</td>
</tr>
</tbody>
</table>
2.3.4 Load Combination 3 – Dead Load & Snow Load

The final load combination considered was that of the dead weight of the roof and the snow load. A major concern with the Breeding Barn’s roof structure is that it cannot support a full snow load. For this reason it’s important to accurately predict the snow load the barn can expect to see. Although this load combination was not actually tested, it was used in order to predict the axial tensions in the truss members if the barn were subjected to a full snow load.

The design snow load for the barn was determined using the Minimum Design Loads for Buildings and Other Structures set forth by ASCE. The ground snow load, \( p_g \), for Shelburne is \( 40 \frac{lb}{ft^2} \). Generally the ground snow load can be modified to determine the flat roof snow load, \( p_f \), based upon the exposure, importance, and thermal factors of the building the snow load is being designed for as shown below.

\[
p_f = 0.7c_e c_s lp_g
\]  \( (2.1) \)

However, in Burlington these factors cannot be applied to the ground snow load. Therefore the flat roof snow load is equal to that of the ground snow load of \( 40 \frac{lb}{ft^2} \).

The flat roof snow load may be reduced by the roof slope factor, \( c_s \), to determine the sloped roof snow load as shown below.

\[
p_s = c_s p_f
\]  \( (2.2) \)

The roof slope factor depends upon the slope of the roof and whether or not the interior of the building is heated. With a roof slope of 32° and the barn kept intentionally
below freezing, the roof slope factor is equal to .72. The final design snow load was therefore determined to be 28.8 \( \frac{lb}{ft^2} \) or roughly 30 \( \frac{lb}{ft^2} \).

2.4. Experimental Data

The data collected during each load combination tested were the acceleration time histories for each wrought iron member of the truss. A sample acceleration time history for member 11E under load combination 1 can be seen in figure 2.4. Each wrought iron member of the truss was given an exciting force causing it to vibrate and thus accelerate as well. These vibrations were excited in three different manners. The three methods of excitation used included plucking the member by hand, striking the member with a rubber mallet, and striking the member with a steel hammer. These three methods were used as an attempt to excite as many of the member’s frequencies as possible.

An event-driven triggering method was not used during the testing at the barn. For this reason not all of the acceleration time histories resemble that shown in figure 2.4. Some of the acceleration time histories collected have a lag time at the beginning of the time history where no acceleration is being detected. This is due to the fact that the collection of the data was triggered manually and there is a delay between triggering and excitement of the truss member. This did not have any effect on the results collected from the acceleration time history data.
Figure 2.4: Sample acceleration time history
2.5. Data Acquisition

The acceleration time histories of the members were collected using G-Links, attached to the member such as in figure 2.5. The G-Links as well as the software used to communicate with them was generously provided by MicroStrain of Williston, Vermont. The G-Link is a wireless transmitter that is able to collect and record acceleration data. The acceleration data can be recorded as bits of information or in a more physically meaningful sense in the form of g’s. Acceleration data was collected in both forms during the testing at the barn. The form in which the acceleration data was recorded did not affect the resonant frequencies that were determined from it. Although the G-Links could collect information on the acceleration for three separate axes, for the purpose of this experiment the acceleration time history for only the axis about which the truss member was being vibrated was collected. The G-Links allowed several acceleration time histories to be collected and then wirelessly downloaded to a laptop at a later date.

Figure 2.5: Example of MicroStrain G-Link installed on truss member
The sampling rate at which the acceleration time history was collected was a critical factor in the acquisition of the data. In order to find the resonant frequencies of the truss members, the acceleration needed to be sampled at a rate of at least twice the expected frequencies in the members. For this reason the axial loads of the truss members were estimated for the load combinations being tested using a 2-D frame analysis program written by Dr. Jeffrey Laible. Using these rough estimated axial loads and the properties of the truss members the resonant frequencies for each member were calculated using the beam vibration equation below.

\[ \omega_j^2 = \frac{EI}{mL^4} + \frac{T}{mL^2} \]

Knowing the range of frequencies expected in the members, the sampling rate of the G-Link could be selected. The approximate frequencies for the members ranged from 4 Hz to 60 Hz depending upon the member and load combination. For this reason a sampling rate of 128 Hz was chosen for the G-Link. A much more thorough development of the beam vibration equation is provided in the next chapter.

The placement of the G-Link along the truss members was an important factor in the accurate acquisition of the data. By placing the G-Link at certain points along the truss member it would be possible that some of the resonant frequencies of the member would not be found. For instance, if the G-Link had been placed mid-span along the member, the even resonant frequencies for that member would not be determined from the acceleration time history. A more detailed analysis of this is provided in the next chapter. For this reason the G-Link was placed as to not lose any resonant frequencies of the member.
CHAPTER 3: ANALYSIS

3.1. Fourier Transform

The Fourier transform is an extension of the Fourier series, and allows the frequency components from signals of non-continuous periodic vibrations to be determined. When the truss members at the Breeding Barn were excited, they vibrated at several different resonant frequencies.

The Fourier series, the basis for the Fourier transform, has shown that a continuous periodic function can be expressed as the summation of an infinite number of sine and cosine terms, known as the Fourier series. A periodic function in the form of a Fourier series may be written as

\[ F(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots + a_n \cos n\omega t \]
\[ b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots + b_n \sin n\omega t \]  

or in the form of

\[ F(t) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos n\omega t + b_n \sin n\omega t\} \]

where \( \omega = \frac{2\pi}{T'} \) and is the continuous frequency of the function and \( T' \) is the period of the time series function, \( F \). The coefficients \( a_0, a_n \) and \( b_n \) for the function \( F(t) \) can be calculated as follows

\[ a_0 = \frac{1}{T} \int_{0}^{T} F(t) \, dt \]  

\[ a_n = \frac{2}{T} \int_{0}^{T} F(t) \cos n\omega t \, dt \]  

\[ b_n = \frac{2}{T} \int_{0}^{T} F(t) \sin n\omega t \, dt \]

The coefficient \( a_0 \) is equal to the average value of the function \( F(t) \).
3.1.1. Discrete Fourier Transform

The most common form of the Fourier transform is the discrete Fourier transform (DFT). The DFT is appropriate when a periodic function, $F(t)$ contains only a finite sample of data points. These sample data points are taken at equally spaced time intervals $\Delta t = \frac{T}{N}$, where $T$ is equal to the time that the data is collected, and $N$ is equal to the total number of data points. The time at which each point of data was collected is expressed as $t_j = j\Delta t$ where $j$ ranges from 0 to $N-1$.

Using the DFT the integrals from equations (3.3 – 3.5) may now be replaced with approximate summations. The coefficients $a_n$ and $b_n$ may now be expressed as

$$a_n = \frac{1}{T} \sum_{j=0}^{N-1} F(t_j) \cos n\omega t_j \Delta t, \text{ for } n = 0, 1, 2, \ldots, \infty$$  \hspace{0.5cm} (3.6)

$$b_n = \frac{1}{T} \sum_{j=0}^{N-1} F(t_j) \sin n\omega t_j \Delta t, \text{ for } n = 0, 1, 2, \ldots, \infty$$  \hspace{0.5cm} (3.7)

where $\omega = \frac{2\pi}{T}$. Another form of this, taking into account the missing factor of 2, may be expressed as

$$F(t_j) = 2 \sum_{n=1}^{\infty} \{ a_n \cos n\omega t + b_n \sin n\omega t \}$$  \hspace{0.5cm} (3.8)

The coefficients of $a_n$ and $b_n$ can be combined using a complex form as

$$C_n = a_n - ib_n$$  \hspace{0.5cm} (3.9)

Euler's relationship is

$$e^{-in\omega t_j} = \cos n\omega t_j - i \sin n\omega t_j$$  \hspace{0.5cm} (3.10)

Substituting eq. (3.6) and eq. (3.7) into eq. (3.9) the following is attained

$$C_n = \frac{1}{T} \sum_{j=0}^{N-1} F(t_j) e^{-in\omega t_j} \Delta t$$  \hspace{0.5cm} (3.11)
Making the substitutions of $t_j = j\Delta t$, $T = N\Delta t$ and $\omega = \frac{2\pi}{T}$ into eq. (3.11) the final expression for the discrete Fourier transform is obtained below

$$
C_n = \frac{1}{N} \sum_{j=0}^{N-1} F(t_j)e^{-2\pi i \left(\frac{n}{N}\right)} \text{ for } n = 0, 1, 2, \ldots, N - 1
$$

(3.12)

along with its inverse discrete Fourier Transform

$$
F(t_j) = \sum_{n=0}^{N-1} C_n e^{2\pi i \left(\frac{n}{N}\right)} \text{ for } j = 0, 1, 2, \ldots, N - 1
$$

(3.13)

3.1.2. Aliasing

The rate at which the acceleration data is collected plays a very important role in how accurate the results of the Fourier transfer are. If any frequency component that occurs in the signal or time history is greater than $\frac{1}{2}$ of the sampling frequency, distortions to the lower frequencies will be generated. These errors occur because higher frequencies cannot be dissociated from lower frequencies because the time interval between data points is too small. A graphical representation of this is shown in figure 3.1.

![Figure 3.1: Example of aliasing](image-url)
In order to avoid the problem of aliasing, the sampling frequency must be at least twice that of the largest frequency expected in the time signal. This frequency is commonly referred to as the Nyquist frequency. The Nyquist frequency can be determined using some basic relationships between the time and frequency domains. The Nyquist frequency is defined as

$$f_{\text{max}} = \frac{N}{2} \Delta f$$

(3.14)

where $\Delta f$ is the lowest frequency component measurable from the time signal and is equal to

$$\Delta f = \frac{1}{T}$$

(3.15)

Substituting eq. (3.15) and $T = N\Delta t$ into eq. (3.14), the Nyquist frequency can be simplified to

$$f_{\text{max}} = \frac{1}{2\Delta t} \text{ Hz}$$

(3.16)

3.1.3. Power Spectral Density

When the DFT is applied to the acceleration time history, the result is the power spectral density function, $C_n$. The power spectral density function describes how the strength of a signal in time is distributed with frequency. An example of a measured acceleration time history of 4 seconds and its respective power spectral density function with a maximum frequency of 64 Hz is shown in Figure 3.2. The peaks on the power spectral density generally represent the resonant frequencies of the truss member. The frequencies can be determined from a visual inspection of the plot. It is important to realize that not every peak on the power spectral density necessary represents a resonant frequency of the truss member being tested. It's possible that vibrations in other members of the truss, or even the entire structure as a whole, can be part of the raw acceleration data. The frequencies at which these vibrate will appear in the power spectral density. It is therefore necessary to have an idea of each member's estimated frequencies beforehand, as well as any other frequencies of the barn that may be measured during the test.

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Figure 3.2: Member 11E excited by hand under load combination # 1; (a) raw acceleration data in time series; (b) power spectral density of acceleration data in frequency domain.
3.2. Beam Vibrations

Consider a pin ended member of length L subjected to axial tension. A typical element of this member can be selected at a distance x along its length with vertical displacement v as shown in Figure 3.3. The beam element is subjected to axial tension T, in addition to positive shear (V) and moment (M) according to the usual beam sign convention. The axial loading on the beam is assumed to be constant in magnitude as well as direction throughout the length of the beam. If the beam maintains a constant cross-sectional area over the entire length and is constructed of one material, the mass density m also remains constant throughout the beam. Considering vertical vibrations of the member, the dynamic equilibrium conditions for forces along the y-axis is

\[ \sum F_y = ma \] (3.17)

Substituting values from figure 3.3 into the above equation

\[ V - \left[ V + \frac{dv}{dx} \right] dx = m \frac{d^2v}{dt^2} dx \] (3.18)

Simplifying eq. (3.18) by canceling like terms V and dx

\[ m \frac{d^2v}{dt^2} = -\frac{dv}{dx} \] (3.19)

Neglecting the rotational mass moment of inertia I, summing moments about the neutral axis on the right side of the beam with values from Figure 3.3

\[ \sum M = 0 \Rightarrow -M + \left( M + \frac{dM}{dx} dx \right) - V dx - T \frac{dv}{dx} dx = 0 \] (3.20)

Canceling like terms and dividing by dx

\[ V = \frac{dM}{dx} - T \frac{dv}{dx} \] (3.21)

Substituting V from eq. (3.21) into eq. (3.19) gives

\[ m \frac{d^2v}{dt^2} = -\frac{d^2M}{dx^2} + T \frac{d^2v}{dx^2} \] (3.22)

30
From basic Euler bending theory

\[ M = EI \frac{d^2v}{dx^2} \]  \hspace{1cm} (3.23)

Since the flexural rigidity (EI) is constant for the entire beam, M from eq. (3.23) can be substituted into eq. (3.22)

\[ m \frac{d^2v}{dt^2} = -EI \frac{d^4v}{dx^4} + T \frac{d^2v}{dx^2} \]  \hspace{1cm} (3.24)
Assuming separable functions sinusoidal in space and time

\[ v(x, t) = V \sin \alpha x \sin \omega t \]  

(3.25)

where \( \alpha \) is angular acceleration and \( \omega \) is the angular frequency leads to

\[ m \frac{d^2}{dt^2} (V \sin \alpha x \sin \omega t) = -EI \frac{d^4}{dx^4} (V \sin \alpha x \sin \omega t) + T \frac{d^2}{dx^2} (V \sin \alpha x \sin \omega t) \]  

(3.26)

Differentiating eq. (3.26) using separation of variables and canceling likes terms

\[ m \omega^2 = EI \alpha^4 + T \alpha^2 \]  

(3.27)

Solving for \( \omega^2 \)

\[ \omega^2 = \frac{EI \alpha^4 + T \alpha^2}{m} \]  

(3.28)

For a simply supported beam, the vertical displacements at each end of the beam must be zero.

Therefore boundary conditions for this problem are

\[ v(t)|_{x=0} = 0 \]
\[ v(t)|_{x=L} = 0 \]  

(3.29)

In order for eq. (3.28) to be accurate these boundary conditions must be true. The first boundary condition is automatically satisfied by the sine function. This is due to the fact that \( \sin 0 = 0 \). The second boundary condition is met only when

\[ \sin \alpha L = 0 \]
\[ \alpha_j = \frac{j\pi}{L} \text{ for } j = 1, 2, ..., \infty \]  

(3.30)

Substituting the required value of \( \alpha_j \) from eq. (3.30) into eq. (3.28)

\[ \omega_j^2 = \frac{EI}{m} \left( \frac{j\pi}{L} \right)^4 + \frac{T}{m} \left( \frac{j\pi}{L} \right)^2 \text{ for } j = 1, 2, ..., \infty \]  

(3.31)

This is simply manipulated to achieve \( \omega_j \) as

\[ \omega_j = \sqrt{\frac{EI \left( \frac{j\pi}{L} \right)^4 + T \left( \frac{j\pi}{L} \right)^2}{m}} \text{ for } j = 1, 2, ..., \infty \]  

(3.32)
Angular frequency \( \omega \) (in radians per second) is related to frequency \( f \) (in Hertz or cycles per second) by the following relationship

\[
\omega = 2\pi f
\]  

(3.33)

Substituting the relationship between angular frequency and frequency from eq. (3.33) into eq. (3.32), the resonant frequencies in Hz are computed as

\[
f_j = \frac{1}{2\pi \sqrt{\frac{EI(\frac{\pi}{L})^4 + T(\frac{\pi}{L})^2}{m}}} \text{ for } j = 1, 2, \ldots, \infty
\]  

(3.34)

From Eq. (3.34) it can be seen that the frequencies, \( f_j \) are dependent upon \( E, I, L, m, T, \) and \( j \). If all of these factors remain constant as stated previously, the only factor that plays a role in \( f_j \) changing is \( j \). It can easily be seen from Eq. (3.34) that as \( j \) increases so does the frequency \( f_j \). Due to the boundary conditions only certain frequencies, the resonant frequencies of vibration can occur in the member. At these resonant frequencies the member will oscillate in normal mode shapes that resemble a standing sinusoidal wave. The fundamental mode of a pin ended member is described by an upward movement of the entire beam. The mode shapes for the first three resonant frequencies of a pin ended member can be seen in Figure 3.4.

The normal mode shapes of a pin ended member play an important role in the placement of the G-Links along the truss members. It is important not to place the G-Link at a node location for the lower resonant frequencies that are being excited during the testing. For example, were a G-Link placed directly in the middle of the truss member, all of the even (that meaning \( j \) is even) resonant frequencies would not be present from its acceleration data.
Figure 3.4: Normal modes of vibration;
(a) fundamental first mode; (b) second mode; (c) third mode
3.3. Parameter Estimation

As was stated previously, the goal of the testing at the barn was to determine the axial loads in the truss members. From the measurements taken at the barn, which included the resonant frequencies and their corresponding j values, the axial loads in the members could be determined. Two different methods were employed to determine the axial loads in the truss members. The simpler of the two methods involved solving for only one parameter from Eq. (3.34) using assumed values of E, I, L, and m. This parameter was simply the axial tension in the member. The second method required solving for two separate parameters. These parameters were the axial tension T and the modulus of elasticity in the member, E.

3.3.1. Single Parameter Estimation

As can be seen from Eq. (3.34) the resonant frequencies of a member are dependent upon E, I, L, m, T, and j. The size, geometry, and material of each truss member at the barn are known. From this information, the rotational mass moment of inertia and the mass density can be computed. The lengths of the truss members are also known. In addition to this, the modulus of elasticity for the wrought iron was determined to be 28000 ksi from lab testing conducted on a sample piece from the barn. From the measurements taken at the barn during testing, the resonant frequencies and their corresponding values of j were also obtained experimentally. Therefore the only remaining unknown factor from eq. (3.34) is the axial tension. Simply manipulating eq. (3.34) and solving for T yields

\[ T_j = \frac{4\pi^2 mf_j^2 - EI(j\pi)^2}{(j\pi L)^2} \text{ for } j = 1, 2, ..., n \]  

(3.35)

where n is the number of resonant peaks from the power spectral density function.
3.3.2. Dual Parameter Estimation

Using a two parameter estimation technique to determine the axial loads in the truss members allowed for a comparison between those determined using the single parameter technique. In addition to this, the two parameter technique also solves for the modulus of elasticity of the member, $E$. Since the modulus of elasticity of the wrought iron was reasonably well known, a comparison with the modulus of elasticity determined from the two parameter technique can be used as a gauge in estimating how accurate the axial load results were.

A least square linear regression was used to determine the two unknown parameters $E$ and $T$. Equation (3.31) which solved for $\omega^2$ was used instead of eq. (3.34) in order to avoid the radical. Setting up eq. (3.31) in matrix form and considering only the first six resonant frequencies of each member there results

$$
\begin{bmatrix}
\frac{I}{m} \left( \frac{\pi}{L} \right)^4 & \frac{1}{m} \left( \frac{\pi}{L} \right)^2 \\
\vdots & \vdots \\
\frac{I}{m} \left( \frac{6\pi}{L} \right)^4 & \frac{1}{m} \left( \frac{6\pi}{L} \right)^2
\end{bmatrix}
\begin{bmatrix}
E \\
T
\end{bmatrix} =
\begin{bmatrix}
\omega_1^2 \\
\vdots \\
\omega_6^2
\end{bmatrix}
$$

or more generically

$$
[A] \{p\} = \{d\}
$$

Sometimes resonant frequencies for members were not believed to be accurate or were entirely missing. In order for these frequencies not to affect the results, a weight $w_j$ was assigned to each frequency. In order to keep this simple a weight of 1 was assigned to any frequency thought to be accurate and included in the analysis, while a weight of 0 was assigned to any inaccurate or missing frequency to be excluded. Considering only the first six resonant frequencies the diagonal weighting matrix $W$ is
In order to do away with the values of $\omega^2$ that are inaccurate or missing, eq. (3.37) is pre-multiplied by the weighting matrix $W$

$$[W][A][p] = [W][d]$$  \hspace{1cm} (3.39)

In order to make the matrix equations symmetric, eq. (3.39) is pre-multiplied by $A^T$

$$[A]^T[W][A][p] = [A]^T[W][d]$$  \hspace{1cm} (3.40)

Finally rearranging and solving for the two unknown parameters

$$[p] = ([A]^T[W][A])^{-1}[A]^T[W][d]$$  \hspace{1cm} (3.41)

which is termed the weighted least squares solution.

The solution for the parameters $E$ and $T$ also has a graphical representation through the application of contour plots. It is possible to plot a three-dimensional surface for each resonant frequency $f_j$ from Eq. (3.34). The values of $I$, $m$, and $L$ are known for each member and held constant, while the parameters $E$ and $T$ are allowed to vary along the $x$ and $y$ axes respectively.

The result is a surface that represents all of the possible frequencies for that particular resonant frequency due to combinations of $E$ and $T$. An example of such a plot for member 11E under load combination 1 is shown in Figure 3.5. A contour can then be selected according to the measured value for that particular resonant frequency. This contour is then projected onto the $xy$ plane as shown in Figure 3.6. This represents all of the combinations of $E$ and $T$ that would result in the frequency that was measured in the barn.
Figure 3.5: Surface produced from Eq. (3.34) with contour at $f_1 = 7.4 \, \text{Hz}$ for member 11E under load combination #1.

Figure 3.6: Contour line at $f_1 = 7.4 \, \text{Hz}$ projected onto the xy plane for member 11E under load combination #1.
Surfaces for each resonant frequency $f_j$ can be created along with their respective contours matching the measurements taken at the barn. The surfaces produced for each additional resonant frequency will have a similar shape, but sit above the previous surface and have increased slope along the axes corresponding to E and T. The change in slope along E and T is due to the value of $j$ in eq. (3.34) being raised to a power in both terms that contain E and T. The term containing E is raised to the 4th power while the term containing T is only raised to the 2nd power. As a result of this surfaces with increasing values of $j$ will have slopes with increasing steepness along E. When the contours from these surfaces are plotted they will intersect each other as a result.

A plot of the surfaces for the first six resonant frequencies created from Eq. (3.34) for member 11E subjected to load combination # 1 is shown in Figure 3.7. The contours for each of these surfaces are also featured in this plot. The resonant frequencies that were measured at the barn and used as the contour values are summarized below in Table 3.1. The resulting plot of the contours from Figure 3.7 projected onto the xy plane is shown in Figure 3.8.

Table 3.1: Measured resonant frequencies taken for member 11E subjected to load combination # 1

<table>
<thead>
<tr>
<th>$j$</th>
<th>$f_j$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.4</td>
</tr>
<tr>
<td>2</td>
<td>15.1</td>
</tr>
<tr>
<td>3</td>
<td>23.9</td>
</tr>
<tr>
<td>4</td>
<td>34.1</td>
</tr>
<tr>
<td>5</td>
<td>45.9</td>
</tr>
<tr>
<td>6</td>
<td>59.2</td>
</tr>
</tbody>
</table>
Figure 3.7: Surfaces for member 11E subjected to load combination #1 and their respective contours

Figure 3.8: Contours projected onto xy plane for resonant frequencies of member 11E subjected to load combination #1
As can be seen, the contours intersect at a common point. Theoretically, they should all intersect at the exact same point. This is because all of the frequencies were measured from the same member under the same loading. Therefore, the $E$ and $T$ should be the same from each resonant frequency. Due to experimental error however, they don't all intersect at exactly the same $T$ and $E$. The parameters $E$ and $T$ can visually be taken from Figure 3.8. These values are the same that would be obtained using the method of least squares described earlier in this section. A benefit of this graphical representation is that frequencies that are not accurate are more easily identified. For example if there was one contour which did not intersect well with the others, it could be concluded that this frequency is likely not accurate and should be excluded from the set.
CHAPTER 4: ROOF STRUCTURE COMPUTER MODELS

Although the main support system of the roof at the breeding barn is referred to as a truss, it is actually a frame. In order to be considered a truss, loads must be applied to panel points or nodes of the truss. At the Breeding Barn, purlins transfer loads to the top chord between panel points, making it necessary to use a frame analysis to analyze the roof structure. This was accomplished using a frame analysis program written by Dr. Jeffrey Laible in MATLAB. More specifically the frame program is used to determine the axial loads in the wrought iron truss members.

4.1. Construction Sequence

There is very little information available regarding the construction of the Breeding Barn. The majority of materials existing at the Shelburne Farms archives are Robertson’s original drawings and blue prints. However, there is almost nothing on the subject of how the barn was actually built. Only one photo exists of the barn while in construction. The photo shown in Figure 4.1 was taken by the Duke of Marlborough when he visited Shelburne Farms in October of 1890. With this piece of the barn’s history missing, it makes it very difficult to accurately represent the structural model of the barn’s roofing system.

Of the existing drawings at the archives though, there exists a framing plan and transverse section taken at the lantern that does not contain any x-bracing. In addition to this there is an updated framing plan and transverse section that does include the x-bracing. This appears to be a modification in how Robertson planned on handling lateral loads on the barn.
The foremost question concerning the construction of the barn is when the x-bracing that extends between the trusses was put in place. The x-bracing was included in the original framing plans by Robertson, but there is no mention as to when they were erected as compared to the rest of the barn. Also, it was observed during testing that under dead loading the x-bracing had a lot of slack in comparison to the additional load combinations. These observations lead to the proposed construction sequence and roof structure models that follow below.

The slack in the x-bracing under only the weight of the roof suggests that the x-bracing was not fully installed until after the truss was erected and the entire roof was completed. This could have been accomplished by first constructing the exterior aisles of the barn, which included the columns atop which the trusses were placed. The x-bracing could then be installed, although not fully tightened, at the connections on top of the columns where the
trusses were going to be placed. The trusses, which were likely pre-assembled, were then lifted and positioned into the connections with the x-bracing causing the columns and aisles walls to slightly push out. The x-bracing would have then been tightened, though not nearly enough to bring the walls back into plum. This construction sequence would explain why the x-bracing takes no loading when subjected to dead load alone, but does appear to take on additional loading afterwards.

4.2 Barn Aisles

In addition to the truss and x-bracing, another component of the barn's roof structure is the aisles that run along its exterior. These aisles are two stories in height and constructed of the same southern pine as the rest of the barn as shown in Figure 4.2. The spacing between the side aisle frames is 12 ft., exactly half that of the spacing between the principal trusses. The roof extending over the aisles is supported with a king post truss at the second floor. These aisles take a portion of the outward thrust of the truss and therefore increase the stiffness of the structure as a whole and reduce the axial loads and therefore stresses in the truss members. To duplicate this effect, the aisles were modeled as equivalent members at the foot of the truss.

The stiffness of the equivalent members was equal to that of the barn's aisle structure. For simplicity, the aisle was modeled as having moment connections at each node location. In order to account for the difference in spacing between the side aisles and the principal trusses, the Young's modulus for the aisles members was doubled. The stiffness of the aisle structure was determined by applying a 1 kip load to the point of the aisle structure where the trusses would transfer any outward thrust (node # 9 from Figure 4.2), and the deflection due to this 1 kip load was then measured. The actual measured deflection at node # 9 in the aisle structure
of the barn due to a 1 kip load was determined to be 2.4 in. This resulted in a stiffness of

\[
\frac{1 \text{ kip}}{2.4 \text{ in}} = \frac{kip}{in} = 0.413 \text{ in}^{-1}
\]

for the aisle structure of the barn.

Axial members were used to model the effects of the barn's exterior aisles. Axial members have a stiffness of

\[
k = \frac{AE}{l}
\]

(4.1)

where \(k\) is the stiffness, \(A\) is the area, \(E\) is Young's modulus of elasticity, and \(l\) is the length of the member. The only parameter that has a set value from Eq. (4.1) is the stiffness of the member, \(k\), which is equal to that of the aisle structure. Two of the remaining three parameters can be given an arbitrary value, which allows for the final parameter to be solved. The equivalent member used to model the aisle structure was given arbitrary values for the Young's modulus of elasticity (\(E\)) and length (\(l\)) of 1600 \(\text{in}^2\) and 100 in respectively. The Young's modulus was chosen to be the same as southern pine used throughout the barn, while the length was truly arbitrary. The area of the equivalent member was then determined to be

\[
A = \frac{kl}{E}
\]

(4.2)

From this it was determined that a member with an area of 0.026 \(\text{in}^2\), Young's modulus of elasticity of 1600 \(\text{in}^2\), and length of 100 in had the same stiffness as the aisle structure. This was chosen as the equivalent member that was used to model the effects of the aisle structure.
Figure 4.2: Structural model of the Breeding Barn's exterior aisles
4.3 Two-Dimensional Model

Given the proposed construction sequence, when the truss is subjected to dead load it's as if the x-bracing in the barn does not exist. Modeling the roof system for this loading scenario is therefore simplified to the combination of the truss and the aisles modeled as equivalent members. Since the truss and the aisles are in plane, a two-dimensional frame works well for modeling the behavior of the roof structure under this loading. The model used for this loading combination, load combination # 0, is shown in Figure 4.3.

![Figure 4.3: Two-dimensional model of Breeding Barn roof structure](image)

Using a two-dimensional frame analysis, the expected axial loads in the truss members tested at the barn can be determined. The axial loads that are predicted with this model are shown below in Table 4.1.

<table>
<thead>
<tr>
<th>Member</th>
<th>11E</th>
<th>11W</th>
<th>12</th>
<th>13E</th>
<th>13W</th>
<th>15E</th>
<th>15W</th>
<th>16E</th>
<th>16W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Load (lb)</td>
<td>7245</td>
<td>7245</td>
<td>8930</td>
<td>7325</td>
<td>7325</td>
<td>3115</td>
<td>3115</td>
<td>3025</td>
<td>3025</td>
</tr>
</tbody>
</table>
4.4 Pseudo Three-Dimensional Model

A three-dimensional model was another model used to attempt and duplicate the behavior of the roof structure of the Breeding Barn. In order to use the two-dimensional frame analysis program previously described, a "pseudo" three-dimensional model was constructed. X-bracing is used to support every truss in the barn. Since the x-bracing runs between the trusses, the loading of one truss will affect the loads experienced in an adjacent truss. This in turn will have an effect on all of the trusses down the line. This is why a two-dimensional frame with an equivalent x-bracing member is not appropriate. To take this into account, five trusses and their respective equivalent members representing the side aisles were used in the pseudo three-dimensional model. This included the truss being tested and two additional trusses to the east and west as shown in Figure 4.4. In a true three-dimensional model the five trusses would all have different coordinates. The x and y coordinates of the trusses would be identical, but the z coordinates would be different for each truss. In order to use a two-dimensional frame analysis the trusses and the x-bracing are pushed together like an accordion, until they're all sitting atop one another. Although in two dimensions, five separate trusses still exist. Each truss is given a unique set of node and element numbers. This allows the trusses to still be connected with x-bracing and loaded separately.
Pushing the trusses together does alter the length of the x-bracing. In three dimensions, the length of the x-bracing is approximately 76 ft. However, in the pseudo three-dimensional model the x-bracing is only 72 ft. in length. In order to account for this the area must be altered in the pseudo three-dimensional model to give the x-bracing the same stiffness as it would have in a true three-dimensional model. The stiffness of x-bracing is given as

$$k_x = \frac{2 \cos \alpha^2 EA}{L}$$  \hspace{1cm} (4.3)

where $\alpha$ is the angle between the x-bracing and the truss, $E$ is Young’s modulus of elasticity of the member, $A$ is the actual area of the x-bracing, and $L$ is the original length of the x-bracing. However, since only one member of the x-bracing is being considered the stiffness is reduced to
\[ K_x = \frac{\cos \alpha^2 EA}{L} \]  \hspace{1cm} (4.4)

As the x-bracing is pushed together in the pseudo three-dimensional model they become axial members with a stiffness of

\[ K = \frac{EA_{eq}}{l} \]  \hspace{1cm} (4.5)

where \( E \) is Young's modulus of elasticity, \( A_{eq} \) is the equivalent area of the x-bracing needed in the pseudo three-dimensional model to keep the required stiffness, and \( l \) is the length of the x-bracing in the pseudo three-dimensional model. Setting Eq. (4.5) equal to that of Eq. (4.4), the necessary area required for the pseudo three-dimensional model \( A_{eq} \) is determined as

\[ A_{eq} = \frac{E}{L} \cdot A \cos \alpha \sp{2} \]  \hspace{1cm} (4.6)

In the pseudo three-dimensional model the x-bracing has a length of 72 ft. in comparison to its actual length of 76 ft. The x-bracing has dimensions of \( \frac{3}{4} \) in x 3 in for an area of 2.25 in\(^2\), while the angle between the x-bracing and the truss is 18°. This results in an equivalent area for the x-bracing of 1.93 in\(^2\). Using the pseudo three-dimensional model the axial loads of the truss members can be predicted as shown below in Table 4.2.

**Table 4.2: Axial loads of truss members tested predicted using pseudo three-dimensional model.**

<table>
<thead>
<tr>
<th>Member</th>
<th>11E</th>
<th>11W</th>
<th>12</th>
<th>13E</th>
<th>13W</th>
<th>15E</th>
<th>15W</th>
<th>16E</th>
<th>16W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Load (lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load Combo # 0</td>
<td>2135</td>
<td>-620</td>
<td>2140</td>
<td>2430</td>
<td>2470</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Load (lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load Combo # 1</td>
<td>3025</td>
<td>-780</td>
<td>3035</td>
<td>3400</td>
<td>3450</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Load (lb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Load Combo # 2</td>
<td>4390</td>
<td>1370</td>
<td>4405</td>
<td>3760</td>
<td>3795</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

50
4.5. Combination of Two-Dimensional & Pseudo Three-Dimension Model

The final model proposal for describing the behavior of the roof structure at the Breeding Barn is the combination two-dimensional and pseudo three-dimensional model. This model was developed to best reflect the proposed construction sequence described earlier in this chapter. This model uses approaches from both the two-dimensional and pseudo three-dimensional models. The axial loads in the truss members due to the dead weight of the roof are determined using the two-dimensional model, which doesn’t take into account the effects of the x-bracing throughout the barn. However, when the roof structure of the Breeding Barn is subjected to loads in addition to the weight of the roof, the x-bracing takes a portion of the additional load. In order to accurately model the structure under these loading circumstances the x-bracing must be included with the truss and the aisles. The axial loads in the truss members due to an additional load applied to the truss is determined using the pseudo three-dimensional model. When doing this, it is important that only the additional load be applied using this model. In order to determine the total axial load in the members, axial loads determined from the pseudo three-dimensional model must be added to axial loads determined from the two-dimensional model designed for the dead load. The axial loads predicted using this model for the additional load combinations are shown in Table 4.3.
Table 4.3: Axial loads of truss members tested predicted with the combination two-dimensional and pseudo three-dimensional model

<table>
<thead>
<tr>
<th>Member</th>
<th>11E</th>
<th>11W</th>
<th>12</th>
<th>13W</th>
<th>15E</th>
<th>15W</th>
<th>16E</th>
<th>16W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (ft)</td>
<td>22.4</td>
<td>22.4</td>
<td>26.25</td>
<td>22.4</td>
<td>22.4</td>
<td>21.7</td>
<td>21.7</td>
<td>21.7</td>
</tr>
<tr>
<td># Rods</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Diameter (in)</td>
<td>1</td>
<td>1</td>
<td>1.5</td>
<td>1</td>
<td>1</td>
<td>.75</td>
<td>.75</td>
<td>.75</td>
</tr>
</tbody>
</table>

**LOAD COMBINATION #0**
Dead Load

Dead Load Prediction (lb)
Determined using two-dimensional model

<table>
<thead>
<tr>
<th>LOAD COMBINATION #0</th>
<th>7245</th>
<th>7245</th>
<th>8930</th>
<th>7325</th>
<th>7325</th>
<th>3115</th>
<th>3115</th>
<th>3025</th>
<th>3025</th>
</tr>
</thead>
</table>

**LOAD COMBINATION #1**
Load Test #1

| LC # 1 Prediction (lb) | 895 | 895 | -150 | 895 | 895 | 970 | 970 | 9804 | 980 |
| LC # 1 + DL Prediction (lb) | 8140 | 8140 | 8780 | 8220 | 8220 | 4085 | 4085 | 4005 | 4005 |

**LOAD COMBINATION #2**
Load Test #2

| LC # 2 Prediction (lb) | 2255 | 2255 | 19901 | 2270 | 2270 | 1330 | 1330 | 1325 | 1325 |
| LC # 2 + DL Prediction (lb) | 9500 | 9500 | 10920 | 9595 | 9595 | 4445 | 4445 | 4350 | 4350 |

**LOAD COMBINATION #3**
Snow Load

| Snow Load Prediction (lb) | 3990 | 3990 | -4350 | 3980 | 3980 | 6030 | 6030 | 6165 | 6165 |
| SL + DL Prediction (lb) | 11235 | 11235 | 4580 | 11305 | 11305 | 9145 | 9145 | 9190 | 9190 |
CHAPTER 5: RESULTS

As previously stated, one of the primary motives for this research was to determine the axial loads of the wrought iron truss members in tension for a range of different loading scenarios. Of particular interest were the in-situ axial loads of the truss members due to the dead load of the roof. Knowing the axial loads in the truss members under this loading was crucial in order to assess safety of the barn as it is used on a daily basis. The axial loads in the truss members for this loading could only be found using the vibration testing technique used throughout this research. Other common methods, such as the use of strain gauges, were not as useful for this project. This is because strain gauges can only detect changes in strain and therefore only changes in loads. However due to the size and weight of the roof at the Breeding Barn, significant axial loads are induced in the truss members. This in-situ axial load due to the dead load of the roof is a significant portion of any additional loading and therefore cannot be ignored.

A loading of further interest was that of a full snow load on the roof. A major concern for the Breeding Barn is that under a full snow load, members of the truss might yield without additional reinforcing. Applying a load to the roof of the barn equivalent to that of a full snow load is obviously not a viable option. Therefore it is necessary to have an accurate model of the barns roof structure to predict loads that can not be safely applied to the barn. This was accomplished by subjecting the barn to a series of load tests that, unlike a full snow load, could safely be conducted. The axial loads experimentally determined from these additional load tests were compared to
several proposed frame models of the Breeding Barns roof structure. From these comparisons it was determined which model of the roof structure best fit the actual in-situ conditions of the truss for the different load combinations. Using this model, it is possible to predict the axial loads in the truss members due to increasing loadings, such as a full snow load, that can not safely be applied to the truss without risking the possibility of some of the truss members yielding.

5.1. Load Combination # 0 – Dead Load

5.1.1. Measured Resonant Frequencies

Acceleration time histories were collected for each of the wrought iron truss members using three different methods of excitation. As stated earlier, the three methods of excitation used included 1) plucking the member by hand, 2) striking the member with a rubber mallet, and 3) striking the member with a steel hammer. These three methods were used as an attempt to excite as many of the members frequencies as possible. The method of excitation appeared to have an effect when determining the resonant frequencies of the member. For example, the power spectral density produced from an acceleration time history created by plucking members by hand tended to have more distinct peaks for the lower resonant frequencies. On the other hand, power spectral densities produced from an acceleration time history created by striking members with a steel hammer tended to have more distinct peaks for higher resonant frequencies. Acceleration time histories created by striking members with a rubber mallet produced power spectral densities with peaks that could be used to identify
lower as well as higher resonant frequencies. Using three different power spectral densities from the three different methods of excitation gave a better overview of the truss member's resonant frequencies as compared to only using one power spectral density from one method of excitation. A sample of a power spectral density for member 11E subject to dead load and excited by hand can be seen in Figure 5.1. Using the three power spectral densities for member 11E subject to dead load, the first six resonant frequencies for the member were determined to be 6.8, 14.2, 22.4, 32.2, 43.2, and 56.6 Hz.

Figure 5.1: Power spectral density of member 11E subject to load combination # 0.
The frequencies due to the dead loading for the remaining truss members were determined by examining their respective power spectral densities as described above. The power spectral densities for the remaining truss members subjected to dead load as well as the other load combinations can be found in Appendix A. The resonant frequencies of the wrought iron truss members in tension due to the dead load of the roof can be seen in Table 5.1.

<table>
<thead>
<tr>
<th>Member #</th>
<th>Resonant Frequency Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>11E</td>
<td>6.8</td>
</tr>
<tr>
<td>11W</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>3.9</td>
</tr>
<tr>
<td>13E</td>
<td>-</td>
</tr>
<tr>
<td>13W</td>
<td>7</td>
</tr>
<tr>
<td>15E</td>
<td>-</td>
</tr>
<tr>
<td>15W</td>
<td>-</td>
</tr>
<tr>
<td>16E</td>
<td>-</td>
</tr>
<tr>
<td>16W</td>
<td>-</td>
</tr>
</tbody>
</table>

On a few occasions, the G-Links did not properly record the acceleration time histories. It is because of this that you will note that some of the members in Table 5.1 don’t have any data. From Table 5.1, you will also notice that some members are
missing only individual resonant frequencies. In this instance power spectral densities were produced for the member, but resonant peaks that are believed to exist for that member were not present on the power spectral densities. When resonant frequencies were missing or believed to be inaccurate they were given a weight of zero as described in section 3.3.2. By weighting these frequencies zero in the weighting matrix, W, the axial loads determined from the parameter estimation techniques were more accurate.

5.1.2. Experimental Axial Loads

With the resonant frequencies for the truss members known, the axial loads of the truss members subjected to the weight of the roof can be determined using the parameter estimation techniques described in section 3.3. These techniques included a single parameter as well as dual parameter estimation. The axial loads determined from these resonant frequencies for the two techniques considered are summarized below in Table 5.2.

In Table 5.2, axial loads for members 11W, 13E, 15W, and 16W are missing. These could not be determined because the resonant frequencies for these members were not determined since the acceleration time histories for these members under this loading were not properly recorded with the G-Links.
Table 5.2: Axial loads of truss members subjected to load combination # 0 determined using single and dual parameter estimation techniques

<table>
<thead>
<tr>
<th>Member #</th>
<th>Axial Load (lb)</th>
<th>Single</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>11E</td>
<td>7007</td>
<td></td>
<td>6898</td>
</tr>
<tr>
<td>11W</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>9906</td>
<td></td>
<td>8368</td>
</tr>
<tr>
<td>13E</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>13W</td>
<td>7614</td>
<td></td>
<td>7288</td>
</tr>
<tr>
<td>15E</td>
<td>2789</td>
<td></td>
<td>2767</td>
</tr>
<tr>
<td>15W</td>
<td>-</td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>16E</td>
<td>3260</td>
<td></td>
<td>3358</td>
</tr>
<tr>
<td>16W</td>
<td>-</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Table 5.2, axial loads for members 11W, 13E, 15W, and 16W are missing. These could not be determined because the resonant frequencies for these members were not determined since the acceleration time histories for these members under this loading were not properly recorded with the G-Links.

The axial load determined using dual parameter estimation can be presented graphically using a contour plot as described in 3.3.2. A sample contour plot for member 11E subjected to dead load is shown below in Figure 5.2. Notice how the contours intersect at a common point. The y coordinate of this point is the axial tension of the member, while the x coordinate is equal to the member's modulus of elasticity. Note that the modulus of elasticity is very close to the determined value of 28,000,000 psi.
Figure 5.2: Contour plot of member 11E when subject to load combination #0
5.1.3. Model Comparison

The experimentally determined axial loads can be compared to the axial loads predicted by the three proposed frame models of the Breeding Barn’s roof structure. Two separate model predictions are available to compare the experimental axial loads under this loading scenario. These models are the Two-dimensional model as well as the Pseudo Three-dimensional model. The third model, which is a combination of the 2-D and Pseudo 3-D, is exactly the same as the 2-D model under dead loading and therefore will not be included. The comparison of these models is summarized below in Table 5.3. The first measured axial load is that determined from single parameter estimation, while the second measured axial load is that determined from the dual parameter estimation.

Table 5.3: Axial loads of truss members when subjected to load combination # 0 compared to those predicted by proposed frame models.

<table>
<thead>
<tr>
<th>Member #</th>
<th>Measured</th>
<th>Axial Load (lb)</th>
<th>2-D</th>
<th>Pseudo 3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11E</td>
<td>7007, 6898</td>
<td>7245</td>
<td>2135</td>
<td></td>
</tr>
<tr>
<td>11W</td>
<td>-</td>
<td>7245</td>
<td>2135</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>9906, 8368</td>
<td>8930</td>
<td>-620</td>
<td></td>
</tr>
<tr>
<td>13E</td>
<td>-</td>
<td>7325</td>
<td>2140</td>
<td></td>
</tr>
<tr>
<td>13W</td>
<td>7614, 7288</td>
<td>7325</td>
<td>2140</td>
<td></td>
</tr>
<tr>
<td>15E</td>
<td>2789, 2767</td>
<td>3115</td>
<td>2430</td>
<td></td>
</tr>
<tr>
<td>15W</td>
<td>-</td>
<td>3115</td>
<td>2430</td>
<td></td>
</tr>
<tr>
<td>16E</td>
<td>3260, 3358</td>
<td>3025</td>
<td>2470</td>
<td></td>
</tr>
<tr>
<td>16W</td>
<td>-</td>
<td>3025</td>
<td>2470</td>
<td></td>
</tr>
</tbody>
</table>
5.2. Load Combination # 1 – Load Test # 1

5.2.1. Measured Resonant Frequencies

For load combination # 1 three separate acceleration time histories were collected, one for each method of excitation, for the truss members. The power spectral densities produced from these acceleration time histories for the truss members subjected to load combination #1 can be found in Appendix A. The resonant frequencies determined for the truss members due to this load combination are summarized below in Table 5.4.

Table 5.4: Resonant frequencies of truss members subjected to load combination # 1.

<table>
<thead>
<tr>
<th>Member #</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>11E</td>
<td>7.4</td>
<td>15.1</td>
<td>23.9</td>
<td>34.1</td>
<td>45.9</td>
<td>59.2</td>
</tr>
<tr>
<td>11W</td>
<td>7.4</td>
<td>15.1</td>
<td>23.9</td>
<td>33.8</td>
<td>46</td>
<td>59.2</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>9.7</td>
<td>-</td>
<td>24</td>
<td>34.5</td>
<td>48.7</td>
</tr>
<tr>
<td>13E</td>
<td>6.7</td>
<td>13.9</td>
<td>22</td>
<td>31.3</td>
<td>42</td>
<td>-</td>
</tr>
<tr>
<td>13W</td>
<td>-</td>
<td>14.7</td>
<td>23.3</td>
<td>32.8</td>
<td>43.9</td>
<td>56</td>
</tr>
<tr>
<td>15E</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15W</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16E</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16W</td>
<td>8.43</td>
<td>17.2</td>
<td>26.1</td>
<td>36.2</td>
<td>47.2</td>
<td>59</td>
</tr>
</tbody>
</table>
It can be noted that the resonant frequencies measured during load combination #1 have generally increased in most of the truss members as compared to load combination #0. This is expected for the truss members that experience an increase in axial tension. A visual representation of this is shown in the power spectral density in Figure 5.3 below. Two sets of acceleration time histories were used to create this power spectral density. Both sets were collected from member 11E; however one set was taken during load combination #0 while the other was collected during load combination #2. It is evident from this plot that the peaks shift to the right towards higher frequencies.

![Power Spectral Density Plot](image)

**Figure 5.3:** Power spectral density for member 11E under load combination #0 and load combination #2.
5.2.2. Experimental Axial Loads

With the resonant frequencies for the truss members subjected to load combination #1 known, it's possible to determine the axial loads in the members. Once again the parameter estimation techniques described in section 3.3 can be used to determine the axial loads of the truss members. The axial loads determined for the truss members subjected to load combination #1 using these parameter estimation techniques are summarized in Table 5.5. The axial load for members 15E, 15W, and 16E are not available because the acceleration time histories were not properly recorded for these members during this load combination.

Table 5.5: Axial loads of truss members subjected to load combination #1 determined using single and dual parameter estimation techniques

<table>
<thead>
<tr>
<th>Member #</th>
<th>Axial Load (lb)</th>
<th>Single</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>11E</td>
<td>8266</td>
<td>8238</td>
<td></td>
</tr>
<tr>
<td>11W</td>
<td>8243</td>
<td>8135</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>11160</td>
<td>8202</td>
<td></td>
</tr>
<tr>
<td>13E</td>
<td>6607</td>
<td>6895</td>
<td></td>
</tr>
<tr>
<td>13W</td>
<td>7357</td>
<td>8069</td>
<td></td>
</tr>
<tr>
<td>15E</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>15W</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>16E</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>16W</td>
<td>6065</td>
<td>6024</td>
<td></td>
</tr>
</tbody>
</table>
Once more the axial loads determined using the dual parameter estimation may be graphically portrayed using contour plots. A contour plot for member 11E under load combination #1 is shown in Figure 5.4. This plot is almost identical to the contour plot for member 11E when subjected to load combination #0, except that the intersection of the contours has shifted up vertically. This shift of the common intersection point reflects the increase in tension experienced in the truss member during this load combination. It's important to note though that the intersection of the contours has not shifted along the x-axis and still remains at approximately 28,000,000 psi. This is significant since the modulus of elasticity of the member should not change with a change in loading. The remaining contour plots for each truss member subject to each load combination can be found in Appendix B.
Figure 5.4: Contour plot of member 11E subject to load combination # 1.
5.2.3. Model Comparison

The axial loads determined for the truss members under load combination #1 can be compared to the axial loads predicted by the three frame models proposed to copy the behavior of the Breeding Barn’s roof structure. The comparison of the axial loads determined from the measured resonant frequencies to those predicted by the frame models is examined below in Table 5.6.

Table 5.6: Axial loads of truss members when subjected to load combination #1 compared to those predicted by proposed frame models

<table>
<thead>
<tr>
<th>Member #</th>
<th>Axial Load (lb)</th>
<th>Measured</th>
<th>2-D</th>
<th>Pseudo 3-D</th>
<th>Combo of 2-D &amp; 3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>11E</td>
<td>8266, 8238</td>
<td>10110</td>
<td>3025</td>
<td></td>
<td>8140</td>
</tr>
<tr>
<td>11W</td>
<td>8243, 8135</td>
<td>10110</td>
<td>3025</td>
<td></td>
<td>8140</td>
</tr>
<tr>
<td>12</td>
<td>11160, 8202</td>
<td>12470</td>
<td>-780</td>
<td></td>
<td>8780</td>
</tr>
<tr>
<td>13E</td>
<td>6607, 6895</td>
<td>10220</td>
<td>3035</td>
<td></td>
<td>8220</td>
</tr>
<tr>
<td>13W</td>
<td>7357, 8069</td>
<td>10220</td>
<td>3035</td>
<td></td>
<td>8220</td>
</tr>
<tr>
<td>15E</td>
<td>-</td>
<td>4350</td>
<td>3400</td>
<td></td>
<td>4085</td>
</tr>
<tr>
<td>15W</td>
<td>-</td>
<td>4350</td>
<td>3400</td>
<td></td>
<td>4085</td>
</tr>
<tr>
<td>16E</td>
<td>-</td>
<td>4220</td>
<td>3450</td>
<td></td>
<td>4005</td>
</tr>
<tr>
<td>16W</td>
<td>6065, 6024</td>
<td>4220</td>
<td>3450</td>
<td></td>
<td>4005</td>
</tr>
</tbody>
</table>
5.3. Load Combination # 2 – Load Test # 2

5.3.1. Measured Resonant Frequencies

Like the loading scenarios before it, three separate acceleration time histories were collected for the truss members subjected to load combination # 2. An acceleration time history for each truss member under this loading was collected using the three methods of excitation discussed earlier. The power spectral densities created from these raw acceleration time histories can be found in Appendix A. The resonant frequency of the truss members that were determined from these power spectral densities are shown below in Table 5.7.

Table 5.7: Resonant frequencies of truss members subjected to load combination # 2.

<table>
<thead>
<tr>
<th>Member #</th>
<th>Resonant Frequency Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>11E</td>
<td>-</td>
</tr>
<tr>
<td>11W</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>4.2</td>
</tr>
<tr>
<td>13E</td>
<td>-</td>
</tr>
<tr>
<td>13W</td>
<td>-</td>
</tr>
<tr>
<td>15E</td>
<td>-</td>
</tr>
<tr>
<td>15W</td>
<td>-</td>
</tr>
<tr>
<td>16E</td>
<td>7.2</td>
</tr>
<tr>
<td>16W</td>
<td>-</td>
</tr>
</tbody>
</table>
The resonant frequencies of the truss members have again increased in comparison to those from previous load combinations. As the loadings have increased throughout the load combinations, so have the resonant frequencies of the truss members. This trend can easily be seen in the plot of Frequency (Hz) vs. Resonant Frequency No. for the three load combinations tested. From this plot in Figure 5.5 the plot of the resonant frequencies for each increasing loading combination of member 11E lies atop the previous loading.

![Graph showing the resonant frequencies of member 11E for different load combinations.](Image)

**Figure 5.5:** Plot of frequency vs. mode number for three load combinations tested.
5.3.2. Experimental Axial Loads

Having determined the resonant frequencies for the truss members for loading combination #2, the axial loads of these members under this loading condition can be calculated. The axial loads for members 11W, 13E, and 16W could not be calculated because their respective acceleration time histories were not recorded properly. The axial loads determined for the truss members when subjected to load combination #2 are contained in Table 5.8.

Table 5.8: Axial loads of truss members subjected to load combination #2 determined using single and dual parameter estimation techniques

<table>
<thead>
<tr>
<th>Member #</th>
<th>Axial Load (lb)</th>
<th>Single</th>
<th>Dual</th>
</tr>
</thead>
<tbody>
<tr>
<td>11E</td>
<td></td>
<td>9318</td>
<td>9395</td>
</tr>
<tr>
<td>11W</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>9982</td>
<td>10836</td>
<td></td>
</tr>
<tr>
<td>13E</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>13W</td>
<td>3576</td>
<td>5292</td>
<td></td>
</tr>
<tr>
<td>15E</td>
<td>3614</td>
<td>3660</td>
<td></td>
</tr>
<tr>
<td>15W</td>
<td>3641</td>
<td>3665</td>
<td></td>
</tr>
<tr>
<td>16E</td>
<td>4400</td>
<td>4472</td>
<td></td>
</tr>
<tr>
<td>16W</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
5.3.3. Model Comparison

With the axial loads of the truss members determined for load combination # 2, a comparison can be made to the axial loads predicted by the three proposed frame models of the barn roof structure. This comparison is made below in Table 5.9.

Table 5.9: Axial loads of truss members when subjected to load combination # 2 compared to those predicted by proposed frame models

<table>
<thead>
<tr>
<th>Member #</th>
<th>Measured</th>
<th>2-D</th>
<th>Pseudo 3-D</th>
<th>Combo of 2-D &amp; 3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>11E</td>
<td>9318, 9395</td>
<td>12455</td>
<td>4390</td>
<td>9500</td>
</tr>
<tr>
<td>11W</td>
<td>-</td>
<td>12455</td>
<td>4390</td>
<td>9500</td>
</tr>
<tr>
<td>12</td>
<td>9982, 10836</td>
<td>16450</td>
<td>1370</td>
<td>10920</td>
</tr>
<tr>
<td>13E</td>
<td>-</td>
<td>12590</td>
<td>4405</td>
<td>9595</td>
</tr>
<tr>
<td>13W</td>
<td>3576, 5292</td>
<td>12590</td>
<td>4405</td>
<td>9595</td>
</tr>
<tr>
<td>15E</td>
<td>3614, 3660</td>
<td>4840</td>
<td>3760</td>
<td>4445</td>
</tr>
<tr>
<td>15W</td>
<td>3641, 3665</td>
<td>4840</td>
<td>3760</td>
<td>4445</td>
</tr>
<tr>
<td>16E</td>
<td>4400, 4472</td>
<td>4670</td>
<td>3795</td>
<td>4350</td>
</tr>
<tr>
<td>16W</td>
<td>-</td>
<td>4670</td>
<td>3795</td>
<td>4350</td>
</tr>
</tbody>
</table>
5.4. Additional Frequencies

The resonant frequency of the truss members were determined by visual inspection of their respective power spectral densities. The individual peaks on the power spectral density were identified as the member’s resonant frequency. It is important to note however, that not every peak on the power spectral density necessarily represents a resonant frequency of the truss member being tested. It is possible that vibrations in other areas of the truss, such as other members or even the truss as whole can be detected in the raw acceleration data recorded. The frequencies at which these outside sources vibrate will appear in the power spectral density. This is why it is necessary to have an idea of each members estimated resonant frequencies ahead of time, as well as other frequencies throughout the barn that may measured during the test.

5.4.1. Pendulum Frequencies

One of the possible outside frequencies that could have been detected while testing the truss members was that of the concrete block suspended from the truss. The frequency of this motion is described by that of a pendulum which is

\[ f = \frac{g}{\sqrt{l}} \]  

(5.1)

where \( g \) is acceleration and \( l \) is then length from which the pendulum is swinging. The concrete blocks were suspended from the purlins spanning the truss. The height of these purlins off the ground, as well as the length of the rope from which they were suspended, ranged roughly from 29 to 50 ft. Therefore any frequency detected for the blocks hanging from the lowest purlins would be approximately 1 Hz, while near .8 Hz.
for the blocks hanging from the apex of the truss. The frequencies for the concrete blocks suspended from the truss are so small that they did not appear in the power spectral densities of the truss members.

5.4.2. Vertical Truss Structure Frequencies

The frequency of the barn as a whole moving up and down during the different loading combinations meant another possible set of frequencies being measured during testing. This meant that the truss was vibrating as a whole structure. The resonant frequencies at which the truss was vibrating changed with the different loading combinations. To determine the resonant frequencies, MATLAB was used to solve the eigenvector problem.

The first step in this process involved determining the stiffness matrix of the truss. This was accomplished by first determining the flexibility matrix of the truss and then taking its inverse. The next step involved determining the mass matrix of the truss under each separate loading combination. In order to simplify this, a lumped mass matrix was used. With the lumped mass matrix, the entire weight of the roof, truss, and applied load was distributed among only the nodes of the truss. With the stiffness matrix of the truss and the three lumped mass matrices for each load combination determined, three separate eigenvalue problems were solved using MATLAB. The final solutions for these problems were the resonant frequencies of the truss moving vertically. The first six resonant frequencies of the truss for each load combination are summarized below in Table 5.10.
Table 5.10: Resonant frequencies of truss structure as a whole.

<table>
<thead>
<tr>
<th>Load Combination No.</th>
<th>Resonant Frequency No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>4.1</td>
<td>5.3</td>
<td>6.1</td>
<td>6.1</td>
<td>10.4</td>
<td>14.7</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>3.5</td>
<td>4.5</td>
<td>5.1</td>
<td>5.1</td>
<td>8.9</td>
<td>12.7</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>3</td>
<td>3.7</td>
<td>5.6</td>
<td>6</td>
<td>8.3</td>
<td>12.7</td>
</tr>
</tbody>
</table>

The higher resonant frequencies for the vertical motion of the truss as a whole were similar to the lower resonant frequencies of the individual truss members. When choosing the resonant frequencies of the truss members, a check was made in order to make sure that these frequencies weren't the ones being identified.
CHAPTER 6: CONCLUSION

The application of vibrations testing to determine the axial loads in the pin-ended truss members at the Breeding Barn was successful. Distinct resonant frequencies were determined for each truss member in tension for all three loading combinations allowing the axial loads to be calculated as well. With the increased loadings applied to the truss, measurable increases in the member’s resonant frequencies were also seen. Interestingly enough, the resonant frequencies for a given loading for the double members were not exactly the same as was expected. Although the frequencies were not exactly the same they were close. Some possibilities for this could be loose joint connections, differential settlements of the truss, or even wind loadings on the barn during testing.

By using vibrations testing, the in-situ axial loads in the truss members due to dead weight of the roof were determined. Due to the weight of the roof at the barn, the portion of axial load due to the weight of the roof is a high percentage of the total axial load. It is because of this that the vibration testing is so advantageous compared to using another common technique in strain gauges. When a strain gauge is installed, the existing level of strain is taken as a base level. Therefore strain gauges are really only measuring the change in strain from the base level. So unless the base level of strain is zero, the in-situ axial strains and loads cannot be determined. As previously stated the weight of the roof induces a significant axial load and therefore strain in the truss members. Between the aforementioned techniques, vibration testing is the only method that will produce in-situ axial loads.
Based on the three proposed frame models it was determined that the combination two-dimensional & pseudo three-dimensional model best predicted what the axial loads in the truss members were over the range of loading combinations tested. Originally it was thought that the side aisles of the barn stiffened the truss and the roof structure as a whole and that the x-bracing was simply for lateral support. Upon further inspection the side aisles do little to stiffen the barn and appear not to have been designed to take any thrust from the truss. However based upon the comparison of the test results to the proposed models, it does appears that the x-bracing takes a portion of any additional load applied to the roof and helps pass it along to adjacent trusses. The axial loads predicted in the truss members using the two-dimensional frame model generally appear to be higher than those actually measured during testing. On the other hand, the axial loads predicted using the pseudo three-dimensional model on its own are generally too low when compared to the actual results.

One of the major fears regarding the Breeding Ban is the possibility of the roof being exposed to a full snow load. The concern of this loading is that some of the truss members might yield and ultimately fail. Obviously this loading cannot be tested by applying a similar load to the roof, such as was done with the load combinations tested in this research. Instead, the combination two-dimensional & pseudo three-dimensional model was used to predict the axial loads in the truss members. The resulting axial loads for this snow load of 30 psf are summarized in Table 6.1. The axial loads predicted by the two other proposed models are also contained in Table 6.1.
Table 6.1: Yield loads of truss members compared to axial loads for a full snow load predicted using proposed frame models.

<table>
<thead>
<tr>
<th>Member #</th>
<th>Allowable</th>
<th>2-D</th>
<th>Pseudo 3-D</th>
<th>Combo of 2-D &amp; 3-D</th>
</tr>
</thead>
<tbody>
<tr>
<td>11E</td>
<td>24335</td>
<td>25805</td>
<td>5550</td>
<td>11235</td>
</tr>
<tr>
<td>11W</td>
<td>24335</td>
<td>25805</td>
<td>5550</td>
<td>11235</td>
</tr>
<tr>
<td>12</td>
<td>54870</td>
<td>31820</td>
<td>6050</td>
<td>4580</td>
</tr>
<tr>
<td>13E</td>
<td>24335</td>
<td>26090</td>
<td>5535</td>
<td>11305</td>
</tr>
<tr>
<td>13W</td>
<td>24335</td>
<td>26090</td>
<td>5535</td>
<td>11305</td>
</tr>
<tr>
<td>15E</td>
<td>13687</td>
<td>11100</td>
<td>8385</td>
<td>9145</td>
</tr>
<tr>
<td>15W</td>
<td>13687</td>
<td>11100</td>
<td>8385</td>
<td>9145</td>
</tr>
<tr>
<td>16E</td>
<td>13687</td>
<td>10765</td>
<td>8570</td>
<td>9190</td>
</tr>
<tr>
<td>16W</td>
<td>13687</td>
<td>10765</td>
<td>8570</td>
<td>9190</td>
</tr>
</tbody>
</table>

The yield stress for the wrought iron used throughout the barn was concluded to be 31,000 psi. This was determined by testing several test specimens. These samples came from a king post truss that used to reinforce one of purlins that was found in the barn. With a known yield stress and the areas of the truss members, the load at which the member would yield was determined. These yield loads can also be seen in Table 6.1. The first thing to take from this is that when using the combination two-dimensional & pseudo three-dimensional model none of the truss members fail. However, if the two-dimensional model were used, several of the truss members would have reached their yield point. Although none of the members fail when using the
pseudo three-dimensional model, using this model could eventually lead to a scenario where the model predicted member axial loads below failure when in reality they may be nearing or already have reached failure. A visual representation of how close a member is to failure can be seen in Figure 6.1. Figure 6.1 is a plot of Resonant Frequency vs. Mode Number for member 11E subject to a full snow load. The combination two-dimensional and pseudo three-dimensional model was used to generate this plot.

![Figure 6.1: Plot of frequency vs. mode number for a full snow load of member 11E.](image-url)
Based upon the vibration testing conducted and the frame model developed to model the roof structure of the barn, it appears as though this particular truss is structurally sound. If the remaining trusses throughout the barn behave the same as this one, the roof structure would appear to be safe as it stands. However, in order to truly make that declaration, more trusses need to be tested.

There are areas of this project that could use further research. One of the early problems experienced during this research was pinpointing the resonant frequencies from the power spectral density. Sometimes there appeared to be several peaks around one area, or it was difficult to decide whether or not smaller peaks were actually a resonant frequency. This made it difficult to assign the resonant frequencies to the proper mode number. The fact that three power spectral densities were available for each truss member because three methods of excitation were used helped alleviate this problem. However, an additional technique to avoid this would be to use two accelerometers on the truss member being vibrated. The idea behind this would be to place one of the accelerometers in the middle of the member and the second one a short distance from the end of the member. By placing the first accelerometer in the middle of the truss member, all of the even resonant frequencies will disappear from the power spectral density produced by it. This can be used in tandem with the power spectral density created from the second accelerometer for which the even resonant frequencies still remain to identify the truss member's resonant frequency and proper mode number.
Another addition that could be made to this research would be to conduct a static’s check at one of the nodes of the truss where the wrought iron members are connected for several load combinations. The resonant frequencies for the compression struts, members 14 and 17, were not successfully determined during the testing at the barn for any of the tested loadings. If all of the axial loads for the truss members coming into the node proved to be in equilibrium, it would be further proof that the vibration testing technique was accurate.

One final recommendation for any future research would be to test the x-bracing adjacent to the truss being tested. Originally the x-bracing was believed to not have any impact on a load applied to the roof or truss. Upon additional inspection though it appears this is not the case. By testing the x-bracing using the vibration methods described in this paper, the combination two-dimensional & pseudo three-dimensional model proposed in this paper could be reinforced.
References


Tierney, Martin. “The Breeding Barn Stabilization and Rehabilitation.”


Appendix A: Power Spectral Densities.

A total of nine truss members, including the double members, were tested during this research. Three different load combinations were used, as well as three different methods for exciting the truss members. This yields a total of 81 possible power spectral densities. Included in this appendix are the 46 successful power spectral densities that were created to determine the resonant frequencies of each truss member for the different loading conditions. The power spectral densities are grouped by the truss member, with a sub plot of each successful power spectral density created.
Member 12 (Hand), No Load

Member 12 (Mallet), No Load

Member 12 (Hammer), No Load

Member 12 (Mallet), Test 1

Member 12 (Hand), Test 2

Member 12 (Mallet), Test 2

Member 12 (Hammer), Test 2

Frequency in Hz.
Member 13W (Hand), No Load

Member 13W (Mallet), No Load

Member 13W (Hammer), No Load

Member 13W (Hand), Test 1

Member 13W (Mallet), Test 1

Member 13W (Hammer), Test 1

Member 13W (Mallet), Test 2
Member 15E (Hand), No Load

Member 15E (Mallet), No Load

Member 15E (Hammer), No Load

Member 15E (Hand), Test 2

Member 15E (Mallet), Test 2

Member 15E (Hammer), Test 2

Power Spectral Density

Frequency in Hz.

Frequency in Hz.

Frequency in Hz.

Frequency in Hz.

Frequency in Hz.

Frequency in Hz.
Member 16W (Hand), Test 1

Member 16W (Mallet), Test 1

Member 16W (Hammer), Test 1

Power Spectral Density

Frequency in Hz.

Power Spectral Density

Frequency in Hz.

Power Spectral Density

Frequency in Hz.
Appendix B: Contour Plots

The contour plots were created as means of visually representing the axial load and modulus of elasticity of the truss members determined using the dual parameter estimation. The 18 contour plots that were successfully created to determine the axial loads of the truss members subjected to the different load combinations are included in this appendix.
E (psi) = 2.856e+007 T (lbs-Least Squares) = 6398 T (lbs, Average) = 7007
f(Hz) = 6.8 14.2 22.4 32.2 43.2 56.6
E = 1 inches L = 22.4 feet

Member & Load Case, Frequencies, & Parameters
11# No Load

T (lbs) vs. E (psi) at different frequencies.
Member & Load Case, Frequencies, & Parameters

11E Test 1

\[ E \text{(psi)} = 2.855 \times 10^7 \]

\[ T \text{(lbs-Least Squares)} = 8236 \text{ T(lbs Average)} = 8266 \]

\[ f(\text{Hz}) = 7.4 \quad 15.1 \quad 23.9 \quad 34.1 \quad 45.3 \quad 59.2 \]

\[ d=1\text{inches L=22.4feet} \]
Member & Load Case, Frequencies, & Parameters

11E Test2

E (psi) = 2.785e+007  T (lbs-Least Squares) = 9395  T(lbs, Average) = 9318

f(Hz) = 0  16  25.2  35.7  47.7  51.1

c=1 inches  L=22.4 feet

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Graph showing the relationship between T (lbs) and E (psi) for different frequencies. The graph includes data points for 16 Hz, 25.2 Hz, 35.7 Hz, 47.7 Hz, and 51.1 Hz.
Member & Load Case, Frequencies, & Parameters

11W Test

\[ E \text{ (psi)} = 2.904 \times 10^7 \text{ T (lbs-Least Squares)} = 9136 \text{ T(lbs, Average)} = 8243 \]

\[ f(\text{Hz}) = 7.4 \quad 15.1 \quad 23.9 \quad 33.8 \quad 46 \quad 59.2 \]

\[ d=1 \text{ inches} \quad L=22.4 \text{ feet} \]
Member & Load Case, Frequencies, & Parameters

12 No Load

\[ E(\text{psi}) = \frac{3.262 \times 10^7 \text{T(lbs-Least Squares)}}{8368 \text{T(lbs, Average)}} = 6906 \]

\[ f(\text{Hz}) = 3.9 \quad 8.6 \quad 15.4 \quad 23.9 \quad 34.8 \quad 48.7 \quad 63 \]

\[ d=1.5\text{inches} \quad L=27.75\text{feet} \]
Member & Load Case, Frequencies, & Parameters

Test 1

- $E (\text{psi}) = 3.288e+007$
- $T (\text{lbs}) = 3202$
- $T (\text{lbs Average}) = 11160$
- $f(\text{Hz}) = 0 \ 9.7 \ 0 \ 24 \ 34.5 \ 48.7 \ 63.1$

$d = 1$ inches, $L = 27.75$ feet

Graph showing the relationship between $T$ (lbs) and $E$ (psi) for different frequencies (3.7 Hz, 24 Hz, 34.5 Hz, 48.7 Hz, 63.1 Hz).
Member & Load Case, Frequencies, & Parameters
12 Test 2
E (psi) = 2.7896e+007 T (Ibs-Least Squares) = 10836 T(Ibs, Average) = 9952
f(Hz) = 4.2  9.21  15.8  25  35.3  47  0

\( d = 1.5\text{ inches} \quad L = 27.76\text{ feet} \)
Member & Load Case, Frequencies, & Parameters
13E Test1

E (psi) = 2.423e+007  T (lbs-Least Squares) = 6895  T(lbs, Average) = 6607
f(Hz) = 6.74  13.9  22  31.3  42  0
d=1 inches  L=22.4 feet

![Graph showing the relationship between E (psi) and T (lbs).]
Member & Load Case, Frequencies, \& Parameters

13W No Load

\( E \text{ (psi)} = 3.081 \times 10^7 \text{T (lbs-Least Squares)} = 7286 \text{T (lbs, Average)} = 7614 \)

\( f(\text{Hz}) = 7 \quad 14.5 \quad 23.3 \quad 33.1 \quad 44.5 \quad 58.4 \)

\( d=1 \text{ inches} \quad L=22.4 \text{ feet} \)

\( T \text{ (lbs)} \) vs. \( E \text{ (psi)} \) graph
Member & Load Case, Frequencies, & Parameters

12W Test 1

$E (\text{psi}) = 2.270 \times 10^7$ $T (\text{lbs, Least Square}) = 8069$ $T (\text{lbs, Average}) = 7357$

$f (Hz) = 0, 14.7, 23.3, 32.8, 43.9, 56$

d = 1\text{ inches } L = 22.4\text{ feet}$

![Graph showing the relationship between $E$ and $T$ for different frequencies.](image-url)
Member & Load Case, Frequencies, & Parameters

13W Test2

\[ E_{\text{psi}} = 1.961 \times 10^7 \text{ T (lbs-Least Squares)} = 5292 \text{T(lbs, Average)} = 3576 \]

\[ f(\text{Hz}) = 0 \quad 0 \quad 19 \quad 0 \quad 37.5 \quad 48 \quad 60.9 \]

d=1 inches L=22.4feet
Member & Load Case, Frequencies, & Parameters

**15E No Load**

\[ E (\text{psi}) = 2.858 \times 10^7 \]

\[ T (\text{lbs-Least Squares}) = 2767 \]

\[ T (\text{lbs, Average}) = 2789 \]

\[ f(\text{Hz}) = 0 \quad 18.5 \quad 26.4 \quad 35.5 \quad 45.8 \quad 57.5 \]

\[ d=0.75\text{inches} \quad L=22.2\text{feet} \]
Member & Load Case, Frequencies, & Parameters
15E Test2
E (psi) = 2.788e+007 T (lbs-Least Squares) =3960 T(lbs, Average) = 3614
f(Hz) = 0 13.2 20.8 29.3 38.8 49.6 61.4
\(d=0.75\) inches \(L=22.2\) feet
Member & Load Case, Frequencies, & Parameters

15W Test2

\( E \text{ (psi)} = 2.783 \times 10^7 \quad T \text{ (lbs, Least Squares)} = 3865 \quad T \text{ (lbs, Average)} = 3641 \)

\( f(\text{Hz}) = 0, 13.4, 21, 29.2, 38.8, 49.6, 61.4 \)

\( d = 0.75 \text{ inches} \quad L = 22.2 \text{ feet} \)

![Graph showing the relationship between force (T) and modulus of elasticity (E) for different frequencies.](image)
Member & Load Case, Frequencies, & Parameters

16F No Load

\[ E (\text{psi}) = 2.678 \times 10^7 \text{ T (lbs-Least Squares)} = 3358 \text{ T(lbs, Average)} = 3260 \]

\[ f(\text{Hz}) = 0 \quad 12.7 \quad 20 \quad 28.1 \quad 37.4 \quad 48 \quad 59.4 \]

\[ d=0.75 \text{inches L=22 feet} \]

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![Graph showing T vs. E for different frequencies.](image_url)
Member & Load Case, Frequencies, & Parameters

16E Test2

\[ E\ (\text{psi}) = 2.670 \times 10^7 \]
\[ T\ (\text{lbs, Least Squares}) = 4472 \]
\[ T\ (\text{lbs, Average}) = 4400 \]

\[ f(\text{Hz}) = 7.2 \quad 14.7 \quad 22.8 \quad 31.7 \quad 41.6 \quad 52.4 \]

\[ d = 0.75\text{inches} \quad L = 22.2\text{feet} \]
Member & Load Case, Frequencies, & Parameters
15W Test 1

$E \text{(psi)} = 2.031 \times 10^7$ $T \text{(lbs-Least Squares)} = 6024 $ $T \text{(lbs, Average)} = 6066$

$f(\text{Hz}) = 8.43$ 17.2 26.1 38.2 47.2 59

d=0.75inches L=22.2feet