Two-Dimensional Numerical Study of Micronozzle Geometry

Jason M. Pearl
University of Vermont

Follow this and additional works at: https://scholarworks.uvm.edu/graddis
Part of the Aerospace Engineering Commons, and the Mechanical Engineering Commons

Recommended Citation
Pearl, Jason M., "Two-Dimensional Numerical Study of Micronozzle Geometry" (2016). Graduate College Dissertations and Theses. 579.
https://scholarworks.uvm.edu/graddis/579

This Thesis is brought to you for free and open access by the Dissertations and Theses at ScholarWorks @ UVM. It has been accepted for inclusion in Graduate College Dissertations and Theses by an authorized administrator of ScholarWorks @ UVM. For more information, please contact donna.omalley@uvm.edu.
TWO-DIMENSIONAL NUMERICAL STUDY OF MICRONEZZLE GEOMETRY

A Thesis Presented
by
Jason M. Pearl
to
The Faculty of the Graduate College
of
The University of Vermont

In Partial Fullfillment of the Requirements
for the Degree of Master of Science
Specializing in Mechanical Engineering

May, 2016

Defense Date: March 30 2016
Thesis Examination Committee:

Darren L. Hitt, Ph.D., Advisor
George F. Pinder, Ph.D., Chairperson
William F. Louisos, Ph.D.
Cynthia J. Forehand, Ph.D., Dean of the Graduate College
Abstract

Supersonic micronozzles operate in the unique viscosupersonic flow regime, characterized by large Mach numbers ($M > 1$) and low Reynolds numbers ($Re < 1000$). Past research has primarily focused on the design and analysis of converging-diverging de Laval nozzles; however, plug (i.e. centerbody) designs also have some promising characteristics that might make them amenable to microscale operation. In this study, the effects of plug geometry on plug micronozzle performance are examined for the Reynolds number range $Re = 80 - 640$ using $2D$ Navier-Stokes-based simulations. Nozzle plugs are shortened to reduce viscous losses via three techniques: one - truncation, two - the use of parabolic contours, and three - a geometric process involving scaling. Shortened nozzle are derived from a full length geometry designed for optimal isentropic performance. Expansion ratio ($\epsilon = 3.19$ and $6.22$) and shortened plug length ($%L = 10 - 100\%$) are varied for the full Reynolds number range. The performance of plug nozzles is then compared to that of linear-walled nozzles for equal pressure ratios, Reynolds numbers, and expansion ratios. Linear-walled nozzle half-angle is optimized to to ensure plug nozzles are compared against the best-case linear-walled design.

Results indicate that the full length plug nozzle delivers poor performance on the microscale, incurring excessive viscous losses. Plug performance is increased by shortening the nozzle plug, with the scaling technique providing the best performance. The benefit derived from reducing plug length depends upon the Reynolds number, with a 1-2% increase for high Reynolds numbers and up to 14% increase at the lowest Reynolds number examined. In comparison to Linear-walled nozzle, plug nozzles deliver superior performance when under-expanded, however, this trend reverses at low pressure ratios when the nozzles become over-expanded.
ACKNOWLEDGEMENTS

I’d like to thank Dr. Hitt and Dr. Louisos for allowing me to pursue my interests and their invaluable guidance and support throughout both undergraduate and graduate studies. Also thank you Debra Fraser, of the Vermont Space Grant Consortium, all your help over the past two years was and is extremely appreciated.

I’d like to thank the Vermont Space Grant Consortium and NASA EPSCoR who supported my studies under NASA Cooperative Agreement NNX13AB35A.
# Table of Contents

Acknowledgements .......................................................... ii  
List of Figures .............................................................. vi  

1 Introduction ............................................................... 1  
1.1 Micropropulsion: 20th Century ...................................... 1  
1.2 Micropropulsion: 21st Century ................................. 4  
1.2.1 Experimental Results ........................................ 4  
1.2.2 Navier-Stokes DSMC Comparisons ......................... 5  
1.2.3 Navier-Stokes Simulations ................................ 7  
1.3 Plug Nozzles .......................................................... 8  
1.3.1 Macroscale ....................................................... 8  
1.3.2 Microscale ...................................................... 9  
1.4 Present Study ......................................................... 10  

2 Geometries ............................................................... 11  
2.1 Overview ............................................................ 11  
2.2 Linear-Walled Designs ............................................ 12  
2.3 Plug Designs ........................................................ 15  
2.3.1 Full Length Plug Contour ................................... 16  
2.3.2 Length Reduction Methods ................................. 17  

3 CFD Model ................................................................. 22  
3.1 Governing Equations ............................................... 22  
3.2 Flow Properties ..................................................... 23  
3.2.1 Propellant Selection ......................................... 23  
3.2.2 Properties of HTP ............................................. 24  
3.3 OpenFOAM .......................................................... 26  
3.3.1 Solver Selection ............................................... 26  
3.3.2 Linear System Solvers .................................... 27  
3.3.3 Numerical Schemes ........................................... 28  
3.3.4 Boundary Conditions ....................................... 28  
3.4 Vetting rhoCentralFoam .......................................... 29  

4 Grid ........................................................................ 31  
4.1 Performance Metrics .............................................. 31  
4.2 Grid Architecture .................................................. 33  
4.3 Grid Generation ..................................................... 35  
4.4 Grid Sensitivity ...................................................... 37
5 Results
5.1 Linear-Walled Nozzles ................................................. 40
5.2 Plug Nozzles ................................................................. 47
5.3 Plug and Linear-Walled Comparison .................................. 61
5.4 Slip Effects ................................................................. 64

6 Conclusion ................................................................. 70
# List of Figures

2.1 Optimal $p_0/p_e$ for the two $\epsilon$ examined. ........................................ 12
2.2 Overview of nozzle geometries examined ..................................................... 13
2.3 Schematic of the linear-walled de Laval nozzle geometry. .............................. 14
2.4 Schematic of the $\epsilon = 6.22$ plug nozzle geometry. ............................... 15
2.5 Isolines of the expander for different Mach numbers. ............................... 16
2.6 Depictions of steps used in the scaling process. ........................................ 18
2.7 Comparison of parabolic and scaled nozzle spike properties ...................... 21
3.1 Properties of fully decomposed 85% HTP. .............................................. 25
3.2 Comparison to past results. .................................................................. 30
4.1 Mesh block architecture. ....................................................................... 34
4.2 Grid before and after mesh solver .......................................................... 34
4.3 Coarse mesh with automated connector spacing labeled. .......................... 36
4.4 Convergence temporally for different mesh sizes ...................................... 39
4.5 Grid sensitivity study results. .................................................................. 39
5.1 Linear-walled nozzle mach contours $\epsilon = 3.19$ ................................. 41
5.2 Linear-walled nozzle mach contours $\epsilon = 6.22$ .................................. 42
5.3 Specific impulse efficiency of linear-walled nozzles ................................. 44
5.4 Specific impulse efficiency of linear-walled nozzles .................................. 46
5.5 Specific impulse efficiency of plug nozzles ............................................... 50
5.6 Increase in specific impulse due to plug length reduction .......................... 51
5.7 Contours of Mach equals 1 at different Reynolds numbers, $\epsilon = 3.19$ .... 52
5.8 Scaled plug nozzle mach contours $\epsilon = 3.19$ ........................................... 53
5.9 Comparison of scaled and parabolic plug nozzle exit states for 60% length, $\epsilon = 3.19$, and $Re = 80$ ............................................................. 54
5.10 Curvature of parabolic and scaled contours ............................................. 55
5.11 Truncated plug nozzle mach contours $\epsilon = 3.19$ and $Re = 80$ ............ 58
5.12 Contours of Mach equals 1 for different length nozzles at $\epsilon = 3.19$ and
    $Re = 80$ .............................................................................................. 59
5.13 Plug nozzle mach contours $\epsilon = 6.22$ and $Re = 80$ ............................ 60
5.14 Specific impulse comparison between linear-walled and plug designs ....... 62
5.15 Mach=1 isolines for a 40% length $\epsilon = 6.22$ plug nozzle. ................. 63
5.16 Contour plots of Knudsen number at $Re = 80$ and 640 ......................... 64
5.17 Wall slip effects on specific impulse for plug $\epsilon = 6.22$ ....................... 66
5.18 The percent increase in specific impulse caused by shortening the full length $\epsilon = 6.22$ plug to 40\% the original length via the scaling method is plotted for slip and no-slip boundary conditions. 68

5.19 Percent difference between slip and no-slip specific impulse for the full length and 40\% scaled $\epsilon = 6.22$ plug nozzles. 69
Chapter 1

Introduction

1.1 Micropropulsion: 20th Century

Over the past half-century the need for micropropulsion has been largely driven by a desire for increased pointing precision in large observational satellites. As the orientation control requirements become more stringent, smaller thrusters are needed to deliver the appropriate impulse bits and thrust levels. In some instances, the thrusters used in these precision Reaction Control Systems (RCS) became small enough to incur significant viscous losses in the nozzle section. Early work, examining the effect of microscaling rocket nozzles occurred in the 1960s and up until the 90s all work in the field was primarily analytic or experimental in nature. In 1966, Sutherland and Maes provided an overview of propulsion systems in the thrust range $10^{-5} - 1 \text{ lbf}$, under development at the time.(1) They discussed “small-nozzle” performance and suggest a potential trade-off between expansion ratio ϵ and expander half-angle α would need to occur to minimize losses due to friction and other factors inherent on the micro-scale.
1.1. MICROPROPULSION: 20TH CENTURY

In 1968, Murch et al. used analytic and experimental methods to examine the effect of expander half-angle and nozzle expansion ratio on the performance of converging-diverging nozzles with throat Reynolds number in range 600-3000 (2). Conical nozzle with half-angle of 10, 20 and 30° degrees were examined as well as horn and bell geometries all with expansion ratio ranging from 1-200. For high Reynolds numbers $\text{Re}^* > 800$ performance increased with increasing expansion ratio, however, Murch suggests that for low Reynolds number this trend might reverse at large expansion ratios. Additionally Murch found the 20° expander cone provided the best performance.

Later, in 1971, Rothe used an electron beam fluorescent technique to study micronozzle flows with throat Reynolds numbers $100 < \text{Re}^* < 1500$ for a conical nozzles with $\alpha = 20^\circ$ (3). For larger Reynolds numbers $\text{Re}^* > 500$ an inviscid core of supersonic flow extend the length of the expander; however, as the Reynolds number decreased, this supersonic core was consumed by the subsonic boundary layer. For Reynolds numbers less than 300 the flow at the exit was fully subsonic, with a supersonic “bubble” existing upstream in the area immediately after the throat. Rothe also noted an increase in center-line temperature at the nozzle exit for low Reynolds number cases and attributed this result to viscous shear stresses converting kinetic flow energy back to thermal energy.

In the 1980s NASA began an initiative to develop a resistojet design for high performance at low Reynolds number. As part of this initiative, researchers at Lewis Research Center sought to build upon past work involving micronozzle design. In 1987 Grisnik et al. examined different expander geometries; one conical nozzle, one bell nozzle, and two trumpet designs for a Reynolds number range of 500-9000 (4).
1.1. MICROPROPULSION: 20TH CENTURY

Unheated \( (T_0 = 300 K) \) molecular nitrogen and molecular hydrogen were used as propellants. All 4 geometries examined had roughly the same expansion ratio and yield roughly equivalent performance levels. That same year, Whalen in her Master’s Thesis with Lewis Research Center, expanded upon Grisnik et al’s study examining multiple expansion ratios 25-200 for cone, bell, and trumpet configurations (5). Multiple expander half-angles were also examined for the cone nozzle configuration with \( \alpha = 15, 20, \) and 25\(^\circ\). The Reynolds number was varied from 150-3000 for hydrogen propellant and 500-6000 for nitrogen. Again the different expander designs performed within experimental error; however, for low Reynolds numbers \( Re < 2000 \), and particularly for large expansion ratios the 25\(^\circ\) conical nozzle appeared to outperform the small half-angle designs as well as the bell nozzle configurations.

Early work in the field of micropropulsion highlights several of the design challenges intrinsic to supersonic micronozzles. At significantly low Reynolds numbers \( Re < 1000 \) viscous effects begin to dominate the flow resulting in reduced performance and in extreme cases fully subsonic flow at the nozzle exit. Additionally, for the low-thrust low-impulse regime of interest, it can be difficult to gain the measurement resolution needed to differentiate the performance of different nozzle configurations. Lastly, analytic techniques fall short in describing micronozzle flow which is influenced by viscous and rarefaction effects.
1.2. MICROPROPULSION: 21ST CENTURY

1.2 MICROPROPULSION: 21ST CENTURY

1.2.1 EXPERIMENTAL RESULTS

Through the 60s, 70s, and 80s, micropropulsion and micronozzle development was driven by a need for small thrust and impulse bits. Small nozzles were needed to deliver impulse bits in a precise manner, however little constraint was placed on the size of the propulsion system as a whole, since the overall satellite was quite large (6). At the turn of the century, interest sparked in reducing spacecraft size, spawning new classes of satellites: microsats (<100kg), nanosats (<10kg), picosats (<1kg), and cubesats (a standardized 10cm cube). These smaller satellite classes were made possible by advances in the field of MicroElectroMechanical Systems (MEMS) and manufacturing which allowed many components and subsystems to be reduced in size and weight (7). Additionally smaller satellites open up new mission architectures (i.e. formation flying) and greatly reduce the cost to access space (8). With the introduction of smaller satellites a constraint on the size of the propulsion subsystem was also introduced, fostering an interest in MEMS-based propulsion system. This led to a shift from axis-symmetric nozzle designs to planar nozzle designs that could more-easily be manufactured using MEMS techniques. A planar nozzle describes a 3D nozzle derived from a 2D pattern. For example, a 2D pattern can be etched into the 3rd dimension to produce a planar nozzle.

In the late 1990s Bayt and Bruer demonstrated experimentally that supersonic flow could be achieved in a planar micronozzle fabricated by etching a silicon wafer (9; 10). They analyzed nozzles with throat widths on the order of 10s of microns and
1.2. MICROPROPULSION: 21ST CENTURY

found that the etch depth of the nozzle played an import role in performance(11; 12).

Higher resolution experimental determinations of thrust, on the sub-micro-Newton scale has been achieved by researchers at the University of Colorado Colorado Springs with the development of a nano-Newton Thrust Stand (nNTS) (13; 14; 15). In 2005, Ketsdever et al. used the nNTS to verify DSMC models for a planar linear-walled nozzle and a sonic orifice at $Re = 0.2$-200 (16). DSMC simulations agreed well with experimental results. Later in 2011, Black, in his M.S. Thesis, examined axis-symmetric conical, bell, trumpet, and plug micronozzle designs with the nNTS (17).

Other microscale experimental work has come out of the Massachusetts Institute of Technology with the development of a micronewton thrust stand by Mirczak in 2003 (18). An updated version of the stand was used by Bruccoleri et al. to examine bell and linear-walled planar micronozzle geometries (19).

1.2.2 NAVIER-STOKES DSMC COMPARISONS

Many of the micropropulsion systems that are of interest, produce very small thrust and impulse bits complicating the characterization of their performance when using experimental setups. As such, a significant body of work has focused on examining micronozzles using numerical techniques. Both continuum based Navier-Stokes (NS) simulation, and rarefied gas models Direct Simulation Monte Carlo (DSMC) have been used to analyze micronozzles. The level of rarefaction is commonly judged by the non-dimension Knudsen number, which is defined as the mean-free path divided by the characteristic length. For Knudsen numbers in the range 0.01-0.1 the flow is often considered to be in the transitional regime. For smaller Knudsen numbers a continuum approach is typically favored due to it’s computational efficiency, however,
1.2. MICROPROPULSION: 21ST CENTURY

for larger Knudsen numbers the continuum assumption breaks down and the DSMC method is preferred (20).

In 2001, Ivanov et al. examined micronozzle performance using the DSMC code SMILE and the NS code GASP (21). For NS simulations the nozzle exit plane was set as the outlet of the computational domain, adversely affecting the accuracy of NS results. As a result, Ivanov et al. recommended including an extended domain past the nozzle exit for NS simulations. Later in 2002, Alexeenko et al. implemented the domain extension suggested by Ivanov et al. in a paper comparing NS and DSMC simulations of micronozzles with a throat Reynolds number of 200 and throat Knudsen number of 0.005 (22). The flow field predicted by NS and DSMC showed differences near the nozzle lip but were in good agreements regarding the thrust output of the nozzle. In 2006, Liu et al. compared DSMC with NS simulations, with and without slip, for a linear-walled de Laval micronozzle test case \((50 < Re < 1000)\) (23). They suggested a local Knudsen number of 0.045 as a bound to Continuum model validity with slip. In 2007, Xie also compared DSMC and NS with slip and used mach contours and mass flow rates to compare results for the Reynolds number range 200-300 (24). An average Knudsen number at or below 0.01 was suggested as the limit of the continuum assumption. Later, in 2011, Torre et al. compared DSMC and NS with slip results for throat Knudsen numbers of 0.008-0.125 (25). Navier-Stokes simulations were found to under-predict the exit velocity and over-predict exit pressure, however, these two effects nearly canceled yielding NS and DSMC thrust measurements within 3%.

Hao et al. compared experimental and NS simulation results for a 20\(\mu m\) throat nozzle and found good agreement in mass flow rate (26). Later papers by Lijo et
1.2. **MICROPROPULSION: 21ST CENTURY**

al., examining shock structure, and Yan et al., examining effects of surface roughness, compared numerical results to the findings of Hao et al.\(^{(27; 28)}\). Again good agreement in mass flow rate was found between DSMC, NS, and experimental results.

Over the last two decades, there has been an effort to identify the range of applicability of the continuum model based on several different Knudsen number metrics: average Knudsen number, throat Knudsen number, and local Knudsen number. The metric used to assess continuum model validity has also varied, from field properties such as mach contours and exit velocity contours to integrated quantities such as mass flow and thrust. In general, the usable Knudsen range of the continuum model is significantly larger when integrated performance quantities are the focus of the study in comparison to local velocity and pressure fields.

### 1.2.3 Navier-Stokes Simulations

In 2001, Hitt et al. examined the development of a High Test Peroxide (HTP) monopropellant MEMS-based microthruster \(^{(6)}\). In subsequent years, a substantial body of work out of the University of Vermont focused on the design and analysis of planar micronozzles for use in the HTP thruster. Louisos and Hitt examined the viscous effects on linear-walled and bell micronozzle performance using 2D and 3D NS simulation\(^{(29; 30; 31)}\). For the Reynolds number range 15-800 viscous wall interactions were found to substantially degrade performance and delay the firing response in transient cases \(^{(32; 33)}\). In a later paper, by Louisos and Hitt, heat transfer effects were also found to have a substantial effect on performance \(^{(34)}\). They also examined the potential for water vapor condensation in the nozzle expander \(^{(35)}\) and later Greenfield et al. looked at the effects that condensed droplets have on
1.3. PLUG NOZZLES

micronozzle performance (36).

Additionally, Hameed and Kafafy et al. have examined the use of resistive heaters as well as a thermoelement heat pump to increase micronozzle performance (37; 38)

1.3 Plug Nozzles

1.3.1 Macroscale

Plug nozzles were originally developed in the mid 1900s for Single Stage To Orbit (SSTO) applications (39; 40). Launch vehicles, during ascent, experience a range of ambient pressures, from approximately 1 atmosphere at launch to vacuum conditions in space. Nozzles are designed to expand the exhaust gas to certain pressure. When the ambient pressure is equal to the designed exhaust pressure the nozzle is said to be optimally expanded, yielding high performance (41). When the ambient pressure is lower the nozzle is under-expanded and thus does not take full advantage of the pressure differential between the chamber and the atmosphere. Conversely when the ambient pressure is higher than the designed exhaust pressure the nozzle is said to be over-expanded and experiences poor performance. In the case of severe over-expansion, the supersonic jet can separate from the nozzle wall due to the adverse pressure gradient yielding potentially dangerous instability. As a result, the potential for flow separation at low altitudes puts a cap on the maximum nozzle expansion ratio limiting performance. Plug nozzles were seen as a potential solution to the separation issue (42). The exhaust flow is oriented inward at the throat and expands by turning against a central body, with the ambient quiescent air providing
1.3. **PLUG NOZZLES**

an artificial external boundary. At low altitudes the exhaust gases reaches the high ambient pressure before reaching the nozzle tip; however, the exhaust jet must still be turned by the plug surface and thus cannot separate. In-depth examinations of large scale plug nozzle performance and the corresponding flow field can be found in Refs. (43; 44; 45).

### 1.3.2 MICROSCALE

Microscale nozzles however, are typically used exclusive in the vacuum of space and as such, the altitude compensating properties of plug nozzles, that make them attractive for SSTO applications, don’t yield a benefit on the microscale. The interest in plug micronozzles designs comes instead from the need to reduces viscous losses. Both planar and axisymmetric plug designs have the potential to decrease the surface area that is in contact with the supersonic flow in comparison to converging-diverging designs. As a result, plug micronozzles have the potential to decrease the performance degradation due to viscous wall interactions.

A few studies have been published examining plug micronozzle performance. Plug nozzle designs have been examined by Zilic et al. for the same operating conditions and general dimensions as the de Laval nozzles of Louisos and Hitt (46). Navier-Stokes results compared well with those of DSMC simulation. Plug truncation was examined and found to be deleterious to plug nozzle performance. Annular plug micronozzle geometries were examined by Stein and Alexeenko, who found shorter plug outperformed longer plug by exploiting pressure thrust and mitigating viscous losses (47). Finally, in 2015 a paper by Giovannini and Abhari examined a method to boost performance by reducing plug nozzle length (48). A linear function was used
to offset the plug surface away from the flow to compensate for the growth of the kinetic boundary layer, yielding better performance.

1.4 **PRESENT STUDY**

The present study builds upon the steady-state NS-based plug micronozzle work of Zilic et al. (46) and the linear-walled micronozzle work of Louisos and Hitt (29) for HTP thrusters. The current work serves two primary purposes: one, to compare different plug design techniques developed specifically for microscale applications, and two, to directly compare the performance of plug and linear-walled designs for the Reynolds number range $Re = 80 – 640$. Three plug design techniques are proposed: one - truncation, two - the use of parabolic profiles, and three - a geometric method involving scaling. All involve shorting the nozzle plug to reduce viscous losses, and are described in detail in Sec. 2.3.2. Plug and linear-walled geometries are then compared at equal Reynolds numbers $Re = 80 – 640$, pressure ratios $PR = 25 – 200$, and area ratios $\epsilon = 3.19$ and 6.22, to examine the merit of using plug designs on the microscale. The effects of velocity slip is also examined.
Chapter 2

Geometries

2.1 Overview

Plug and linear-walled geometries have been considered in the past for use in HTP thrusters (32), but these designs used different expansions ratios $\epsilon$ making a direct comparison of performance difficult. As such, in this study, linear-walled designs and plug designs are generated to have equivalent throat widths, and expansion ratios. By maintaining throat width constant across all geometries, different designs will experience the same throat Reynolds number for a give set of boundary conditions (i.e. $P_0, T_0, P_\infty$). The same expansion ratio is used so that the nozzles being compared will experience optimal expansion (i.e. $P_e = P_\infty$) at the same pressure ratio and thus will exhibit the same over/under-expanded characteristics at off design pressure ratios.

In this study, two expansion ratios are considered for both plug and linear-walled nozzles, 3.19 and 6.22. From quasi-1D inviscid theory, nozzles with $\epsilon = 3.19$ experience optimal expansion at a pressure ratio of $p_0/p_e = 20$ while nozzles with $\epsilon = 6.22$ experience optimal expansion at a pressure ratio $p_0/p_e = 54$. Quasi-1D analyses are
dependent upon the propellant characteristics. The prior results are for fully decomposed, 85% pure, high test peroxide with a specific heat ratio of $\gamma \approx 1.3$. In this study, pressure ratio of $p_0/p_e = 25$, 50, 100, and 200 are examined. As a result, the $\epsilon = 3.19$ nozzles is under-expanded for the full $p_0/p_e$ regime examined and the $\epsilon = 6.22$ nozzles is over-expanded at $p_0/p_e = 25$, nearly optimally-expanded at $p_0/p_e = 50$ and under-expanded at $p_0/p_e = 100$ and 200. The optimal pressure ratios for both the expansion ratios considered are plotted in Fig. 2.1. The pressure ratio range examined in this study is boxed in blue. An overview of the geometries examined in this study is present in Fig. 2.2.

2.2 LINEAR-WALLED DESIGNS

Linear-walled designs are simple to manufacture and provide satisfactory performance on the microscale and are thus a popular option. A schematic of the linear-walled geometry analyzed in this study is shown in Fig. 2.3. The upstream geometry is held
2.2. LINEAR-WALLED DESIGNS

constant with an approximately 12:1 inlet area ratio and 45° converging angle walls. The throat width is held constant at $90\mu m$.

The geometry of the nozzle expander, the region downstream of the throat, is defined by the nozzle half-angle $\alpha$ and the expansion ratio $\epsilon$. For 2D nozzle, the expansion ratio defines the exit width according to:

$$\epsilon = \frac{l_e}{l^*}$$  \hspace{1cm} (2.1)

in which, $l_e$ is the width of the nozzle exit and $l^*$ is the width of the nozzle throat. The length of the expander region is then controlled by $\alpha$ for a given expansion ratio nozzle. In Fig. 2.3, the expander wall for 3 nozzle half-angles ($\alpha = 15, 30, \text{ and } 45$) is shown. Nozzles with larger half-angles are shorter with a smaller surface area in contact with the flow. In this study, linear walled configurations with $\epsilon = 3.19$ and 6.22, and $\alpha = 15, 20, 25, 30, 35, 40, 45, \text{ and } 50^\circ$ are considered.
2.2. LINEAR-WALLED DESIGNS

Figure 2.3: Schematic of the linear-walled de Laval nozzle geometry.
2.3. PLUG DESIGNS

The full length plug contour is designed using a method developed by Angelino, which utilizes the Prandtl-Meyer expansion function (49). All reduced length geometries are then derived from the full length Anglino geometry for a given expansion ratio. Across the geometric design study, throat width $l^* = 90\mu m$ and inlet area ratio $l_{in}/l^* \approx 12$ are held constant. A schematic of the $\epsilon = 6.22$ plug nozzle can be seen in Fig. 2.4. The red box highlights the plug region which is varied throughout the study. Three methods of reducing plug length are considered: 1) truncation, 2) a parabolic contour, and 3) a geometric transformation involving scaling.

Figure 2.4: Schematic of the $\epsilon = 6.22$ plug nozzle geometry.
2.3. PLUG DESIGNS

![Figure 2.5: Isolines of the expander for different Mach numbers.](image)

2.3.1 FULL LENGTH PLUG CONTOUR

From Angelino’s method the plug contour can be determined from the exit mach number \( M_e \), and the specific heat ratio of the working fluid \( \gamma \). In this study, plug nozzle geometries are designed for comparison to linear walled geometries with a specific expansion ratio \( \epsilon \). As such the exit mach number used to design the contour via Angelino’s method must be determined from the desired expansion ratio using Eqn. 2.2.

\[
\epsilon(M_e) = \frac{1}{M_e} \left[ \frac{2}{\gamma + 1} \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right) \right]^{\frac{\gamma + 1}{2(\gamma - 1)}} \tag{2.2}
\]

The Mach angle and the Prandtl-Meyer expansion function can then be assessed for \( M = 1 - M_e \) according to Eqns. 2.3 and 2.4 respectively. Using the Mach angle and Prandtl-Meyer expansion function for this range of Mach numbers, x-coordinates \( P_x \) and y-coordinates \( P_y \) can be determined that define a plug nozzle surface that delivers isentropic expansion of a inviscid gas with constant \( \gamma \).
2.3. **PLUG DESIGNS**

\[ \mu = \arcsin \left( \frac{1}{M} \right) \]  

\[ \nu = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \arctan \left( \sqrt{\frac{\gamma - 1}{\gamma + 1} \left( M^2 - 1 \right)} \right) - \arctan \left( \sqrt{M^2 - 1} \right) \]  

\[ P_x = l^* M \epsilon(M) \cos \left[ \mu(M) - \nu(M) \right] \]  

\[ P_y = l^* M \epsilon(M) \sin \left[ \mu(M) - \nu(M) \right] \]

The functions \( \epsilon(M), \mu(M), \) and \( \nu(M) \) are assessed for \( M = 1 - M_e \). The line that connects the nozzle lip to the point defined by the coordinates \( P_x \) and \( P_y \) is an isoline of constant flow properties (i.e. \( M, P, T, U, \rho \) etc...). Four examples of these isolines are displayed in Fig. 2.5 for \( M = 1, 1.7, 2.4, \) and 3.1.

### 2.3.2 Length Reduction Methods

The full length plug nozzle designed by Anglino’s method results in a curve that is long and flat near the tip. For microscale flows, this long flat region can incurs significant viscous losses without delivering much productive thrust. As a result three methods of shortening the plug are examined with the aim of removing the inefficient elongated section near the tip. The amount the nozzle is shortened is quantified by dividing the length of the shortened plug contour by the that of the full length Angelino plug contour. This is the percent length (%L) of the reduced length nozzle.
2.3. PLUG DESIGNS

Figure 2.6: Depictions of steps used in the scaling process.
2.3. **PLUG DESIGNS**

**Truncation**

Truncated nozzles are created by cutting off a portion of the plug tip to achieve the desired percent length. A flat base then extends from the cut contour to the symmetry plane of the nozzle. The plug can be truncated all the way to the throat. Additionally, in the limit as percent length goes to 100\%, the original Angelino curve is recovered. For the $\epsilon = 3.19$ plug truncation percent lengths of 10, 20, 30, 40, 50, and 60\% are examined. For the $\epsilon = 6.22$ plug truncation percent lengths of 20, 30, 40, 50, and 60\% are examined.

**Parabolic**

Parabolic nozzles are created by using a parabolic contour to connect the nozzle throat to the reduced length nozzle tip. The parabola segment is fully defined by the x-y-coordinates at the throat, the slope $dx/dy$ at the throat, and the x-y coordinates of the nozzle tip. For a given expansion ratio, the slope and location of the throat are constants and as such the parabolic profile of the nozzle is governed by the location of the nozzle tip which is a function of the nozzles percent length ($\%L$). An equation for the parabola can be found by assessing Eqn. 2.7 for the prior 3 mentioned constraints to form the 3-by-3 linear system of equations shown in Eqn. 2.8.

$$ay^2 + by + c = x$$  \hspace{1cm} (2.7)

$$\begin{bmatrix} y^2 & y^* & 1 \\ 0 & 0 & 1 \\ 2y^* & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x^* \\ x_e \\ dx/dy \end{bmatrix}$$  \hspace{1cm} (2.8)
Reduced length nozzles created using a parabolic profile have a lower limit; they cannot be shortened indefinitely like truncated nozzles. The shortest achievable parabolic nozzle, (that comes to a point) is defined by a linear profile (i.e. $a = 0$) with slope equal to $dx/dy|_a$. Shorter parabolic nozzles are mathematically possible but would yield a rounded tip. The reversed concavity would induce flow separation rendering the design unsuitable for precision propulsion applications. For the $\epsilon = 3.19$ nozzle this limit occurs for a percent length of approximately 30%. For the $\epsilon = 6.22$ nozzle it occurs slightly under 20%. As a result parabolic nozzles of 30, 40, 50, and 60 % length are considered for the $\epsilon = 3.19$ nozzle and 20, 30, 40, 50, and 60 % for the $\epsilon = 6.22$ nozzle. Additionally, unlike truncated nozzles the original full length plug is not achieved in the limit of percent length going to 100% (i.e. the Angelino contour is not parabolic).

**Scaled**

Scaled nozzles are created using a series of geometric transformation pictured in Fig. 2.6. The process begins by scaling the Angelino plug contours on either side of the nozzle by a constant factor using the point where the curves meets the nozzle throats as a datum. In Fig. 2.6 a scaling factor of 1.2 is selected as an example. The intersection of these two scaled curves is then taken and used as the new scaled nozzle, pictured in gray in the bottom of Fig. 2.6. The desired length percentage for the shortened nozzle is achieved iteratively.

Nozzles designed by the scaling method have several desirable properties. Similar to the parabolic method, this method preserves the smoothness of the contour at the throat (i.e. the slope remains continuous). The curves shortened by scaling never
2.3. **PLUG DESIGNS**

cross over the original Angelino plug, similar to a set of successive smaller nesting dolls. The parabolic curves on the other hand do breach the original Angelino curve, which can be seen in Fig. 2.4. Fig. 2.7 displays two plots comparing arc length and the tip angle of parabolic and scaled geometries for percent lengths ranging from 20-60%, and $\epsilon = 6.22$. It can be seen that both geometries have fairly similar arc lengths, with the parabolic nozzles being slight shorter. This indicates, for a given percent length nozzle, both geometry types should incur similar viscous loses. The scaled nozzle however, has a smaller tip angle (the angle of the plug curve at the tip) suggesting the scaled class of nozzles might deliver better flow alignment for a given percent length.

Similar to parabolic nozzles scaled nozzles have minimum percent length that is achievable defined by the same linear curve that limits parabolic nozzles. In the case of scaled plugs the curve is achieved by an infinite scaling factor for the original curve. In the limit of a 100% length scaled nozzle the original Angelino curve is recovered (i.e. scaling factor equal to 1).

![Comparison of parabolic and scaled nozzle spike properties](image)

*Figure 2.7: Comparison of parabolic and scaled nozzle spike properties*
CHAPTER 3

CFD Model

3.1 Governing Equations

The conservation equations of mass, momentum, and energy are employed to model the compressible nozzle flow (Eqns. (3.1), (3.2), and (3.3), respectively). The system of equations is closed by the ideal gas equation of state (Eqn. (3.6)) and assumes a frozen flow of decomposed HTP as per Eqn. (3.7). A Newtonian viscous model is used for the viscous stress tensor ($\tau$) as shown in Eqn. (3.4). A continuum flow model is used in this study and has shown a high degree of agreement with rarefied gas models and experimental results for the range of Reynolds numbers and Knudsen numbers examined (16; 46; 22). Continuum models are computational efficient in comparison to rarefied gas models and as such they have been employed in a number of previous studies examining micronozzle performance and design (32).

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0$$  (3.1)
3.2. FLOW PROPERTIES

\[
\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{UU}) = -\nabla p + \nabla \cdot \mathbf{\tau} \quad (3.2)
\]

\[
\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho E \mathbf{U}) = -\nabla \cdot (k \nabla T + (\mathbf{\tau} \cdot \mathbf{U})) \quad (3.3)
\]

\[
\mathbf{\tau} = \mu((\nabla \mathbf{U} + \nabla \mathbf{U}^T) - \frac{2}{3} \nabla \cdot \mathbf{U}) \quad (3.4)
\]

\[
E = h - \frac{p}{\rho} + \frac{U^2}{2} \quad (3.5)
\]

\[
p = \rho RT \quad (3.6)
\]

3.2 FLOW PROPERTIES

3.2.1 PROPELLANT SELECTION

For chemical based thrusters, there are three major fuel options available: 1) solid propellant, 2) liquid bipropellant, and 3) liquid monopropellant (41). Liquid monopropellants offer distinct advantages for microscale propulsion systems. On the microscale, turbulent mixing does not occur as it does on the macroscale, and as such, bipropellants are not an option. Monopropellants avoid the mixing issues inherent to microcombustion. Additionally, unlike solid propellants, monopropellants are capable of start-stop operation and are thus appropriate for an attitude and station-keeping propulsion system (6; 50). Typically, monopropellants such as High Test Peroxide
3.2. *FLOW PROPERTIES*

(HTP) and hydrazine are decomposed in a catalyst bed to produce exhaust gases
for delivery to a supersonic nozzle (6). HTP, a concentrated solution of hydrogen
peroxide and water, has the added advantage of being a “green” fuel and has received
significant attention over the last decade (6; 50).

The simulations conducted in this study use fully decomposed 85% pure HTP
diluted with water as the working fluid. The catalyzed reaction is highly exothermic
and proceeds according to Eqn. (3.7).

\[ 6H_2O_2 + 2H_2O \rightarrow 8H_2O + 3O_2 + \text{heat} \]  \hspace{1cm} (3.7)

3.2.2 Properties of HTP

The use of HTP propellant is implemented in simulation by defining the thermo-
physical properties of the working fluid. For the N-S based simulation, this means
defining the molecular weight \( MW \), the specific heat \( c_p \), the viscosity \( \mu \), and the ther-
mal conductivity \( \kappa \) of the simulated fluid. The molecular weight is set to 22.19 g/mol
and is found by averaging the individual molecular weights of the products of Eqn.

3.7. For temperature range under consideration, The thermal conductivity of fully
decomposed HTP can be treated as a constant at 0.0254 \( W/(m\cdot K) \). The specific heat
and viscosity, however, exhibit substantial dependence upon the temperature of the
flow and thus necessitate a temperature dependent model. A plot of viscosity and
specific heat is presented in Fig. 3.1. Over the temperature range under considera-
tion (100 – 2000K) the dynamic viscosity of decomposed HTP is linear function of
temperature while the specific heat exhibits significant non-linearity.

In this study, thermophysical properties are modeled as temperature dependent
3.2. FLOW PROPERTIES

Figure 3.1: Properties of fully decomposed 85% HTP.
3.3. OPENFOAM

polynomials: \( \kappa \) a \( 0^{th} \) order polynomial (i.e. constant), \( \mu \) as a \( 1^{st} \) order polynomial (i.e. linear), and \( c_p \) as a \( 6^{th} \) order polynomial which was found to provide a good fit for the curve in Fig. 3.1. In preliminary simulations, it was found that portions of the plume, in the spike region, reach temperatures below 300K which necessitates proper modeling of the non-linear low temperature portion of the \( c_p \) curve.

3.3 OPENFOAM

OpenFOAM® (Open source Field Operation And Manipulation) is an open-source CFD tool box, which uses a Finite Volume Method (FVM) to solve continuum mechanics problems. A wide variety of solvers are included in the standard OpenFOAM® package, all designed for different applications: from lapalcianFoam for thermal diffusion problems, to solvers suitable for combustion problems such as rhoReactingFoam. A full list of OpenFOAM® solvers is provided by CFD Direct and can be readily accessed online. OpenFOAM® version 2.2.0 is used in the present studies.

3.3.1 SOLVER SELECTION

With respect to supersonic micronozzle flow, OpenFOAM®’s compressible flow Navier-Stokes (NS) solvers are of interest. Several solvers fall into this category and include: rhoCentralFoam, sonicFoam, rhoSimpleFoam, and rhoPimpleFoam. Three of the solvers, sonicFoam, rhoSimpleFoam, and rhoPimpleFoam are pressure based, while rhoCentralFoam is density based. As a brief overview of each solver, rhoSimpleFoam utilizes the popular SIMPLE algorithm for steady-state compressible flow problems; sonicFoam is a transient trans-sonic/supersonic solver that utilizes a PISO
3.3. **OPENFOAM**

algorithm; rhoPimpleFoam is another transient solver that merges PISO and SIMPLE algorithms; and rhoCentralFoam is a KT/KNP solver that uses the central-upwind schemes proposed by Kurganov and Tadmor (51) or Kurganov et al.(52).

Of the four solvers, a larger body of past work has examined the capabilities of rhoCentralFoam and sonicFoam. In 2009, Greenshields et al. examined benchmark CFD cases using rhoCentralFoam include a shock tube and forward facing step. From the study they concluded that rhoCentralFoam can provide results comparable to popular ROE solvers (53). In 2012, Marcantoni et al. compared the performance of rhoCentralFoam and sonicFoam using a a set of 2D supersonic flow test cases, including flow over a wedge, a diamond airfoil, and a blunt body. They found that sonicFoam required a significantly larger mesh (up to 3x the number of cells) to achieve results equivalent to rhoCentralFoam (54). In light of the past work with the rhoCentralFoam solver, it is selected for use in the present study.

### 3.3.2 Linear System Solvers

OpenFOAM® allows the user to specify the solver used to solve the linear systems of equation (i.e. $Ax = b$) as well as any preconditioning or smoothing methods. Preconditioned solvers solve the equation $M^{-1}Ax = M^{-1}b$ and can achieve quicker convergence. In this study, a preconditioned conjugate gradient (PCG) solver with diagonal preconditioning is used. The absolute tolerance of the solver is set to $1(10)^{-9}$ with a relative tolerance of $1(10)^{-4}$.
3.3. OPENFOAM

3.3.3 NUMERICAL SCHEMES

Central upwind schemes are used by rhoCentralFoam to discretize convective terms of the governing equations. The user has the option to select between the central upwind scheme of Kurganov and Tadmor (KT) (51) or that of Kurganov, Noelle, and Petrova (KNP)(52). The two methods vary in their weighting, $\alpha$ in Eqn. 3.8 during interpolation. The KT method uses and equal weighting (i.e. $\alpha = 0.5$) between the inflow and outflow direction and is therefore a pure central scheme. The KNP method varies $\alpha$ based on the local propagation speed yielding an upwind biased central scheme. Past work has found the KNP method to be more accurate and it is therefore used in this study (52; 53).

$$\sum_f \phi_f \psi_f = \sum_f [\alpha \phi_{f+} \psi_{f+} + (1 - \alpha) \phi_{f-} \psi_{f-} + \omega_f (\psi_{f-} - \psi_{f+})] \quad (3.8)$$

The Total Variation Diminishing (TVD) van Leer limiter is used as an interpolation scheme (55). Past work has shown that the van Leer limiter provides good results when paired with the central upwind schemes of KT and KNP (53; 54).

3.3.4 BOUNDARY CONDITIONS

At the inlet, total temperature and total pressure are specified. The total inlet temperature is set to 886 K, the adiabatic flame temperature of 85% HTP (6). Inlet pressure is varied to achieve different throat Reynolds numbers. Inlet pressures of 25, 50, 100, and 200 kPa are examined corresponding to Reynolds numbers of 80, 160, 320, 640. Quasi-1D theory is used to determine the Reynolds number from the inlet
3.4. VETTING RHOCENTRALFOAM

pressure using Equation 3.9 (56).

\[ Re_t \equiv \frac{h_t p_0 \gamma}{\mu} \sqrt{\frac{(\frac{2}{\gamma+1})^{\frac{\gamma+1}{\gamma-1}}}{\gamma R T_0}} \]  

(3.9)

An ambient pressure of 1.0kPa and ambient temperature of 300K is imposed at the outlet to ensure the validity of the continuum model (46; 32; 57; 29). Continuum simulations with the prescribed outlet condition have agreed well with rarefied gas models and experiments (16; 46; 22; 58). A no slip condition is imposed at the wall assuming continuum flow with negligible rarefaction effects (29). The walls are treated as adiabatic assuming negligible heat flux between the flow and the nozzle substrate. The typical firing time for microthrusters is substantially lower than the characteristic time of heat transfer in the plug substrate validating this approach (34). The effect of velocity slip at the wall is also examined for a subset of geometric design space. A tangential momentum accommodation coefficient (TMAC) of 0.85 is imposed using a 1st order Maxwell slip model consistent with experimental results for polished silicon (59).

### 3.4 VETTING rhoCENTRALFOAM

Density based solvers such as rhoCentralFoam can give convergence issues in low speed regions of the flow. For supersonic nozzles this can result in instability or convergence issues in the nozzle inlet or regions of the quiescent flow around the plume in the extended domain region. To assess rhoCentralFoam ability to model supersonic micronozzle flow, results from W.F. Louisos and D.L. Hitt’s publication “Viscous Effects on Performance of Two-Dimensional Supersonic Linear Micronozzles” (29) is
3.4. VETTING RHOCENTRALFOAM

used as a baseline for comparison. The 30° linear-walled NASA-GSFC micronozzle analyzed in ref (29) is also examined in this study and as such represents a prudent case to test rhoCentralFoam capabilities in the viscous supersonic flow regime. Line plots of pressure and velocity across the nozzle exit are compared between the rhoCentralFoam-OpenFOAM model in red, and that of ref (29) in black for a throat Reynolds number of 800. The x-axis shows distance from the nozzle center-line for which $x = 0$ is the nozzle center-line and $x = 280\mu m$ is the nozzle wall. The models show a high level of agreement in the core of the flow and diverge slightly near the wall. Comparing thrust output, rhoCentralFoam predicts a thrust of 32.83 N/m and the model of Louisos and Hitt predicts a thrust of 33.14 N/m, yielding a percent difference of 0.9%. These results are close enough to attribute the discrepancies to differences in mesh refinement and differences in the methods used to model the thermophysical properties of HTP.

![Figure 3.2: Comparison to past results.](image-url)
4.1 Performance Metrics

Specific impulse and specific impulse efficiency are used as metrics to compare the performance of different nozzle geometries. Specific impulse is the ratio of thrust to sea-level weight flow rate of propellant. It can be considered a form of fuel efficiency metric; high specific impulse engines will generate more thrust per unit mass consumed, yielding longer life-spans for satellites and/or requiring less fuel. Specific impulse is presented in Eqn. 4.1.

\[ I_{sp} = \frac{F}{\dot{m} g_0} \]  

(4.1)

It can be seen that the specific impulse depends upon the thrust \( F \), mass flow rate \( \dot{m} \), and the acceleration due to gravity at sea level \( g_0 \). The thrust and mass flow from continuum simulation are calculated via equation 4.2 and 4.3 respectively. Integrals are taken across the nozzle exit plane (41; 60).
4.1. PERFORMANCE METRICS

\[ F = \int_A \rho U (U \cdot n) \, dA + \int_A (p - p_\infty) n \, dA + \int_A (\tau \cdot n) \, dA. \] (4.2)

Thrust is computed by summing three effects represented by the three surface integrals on the right hand side of Eqn. 4.2. From left to right, first there is the contribution due to momentum flux across the exit plane of the nozzle, second the contribution due to a pressure imbalance with the surrounding medium, and last the contribution due to the fluid stress state at the nozzle exit (which can become appreciable at low Reynolds numbers). The mass flow rate is then calculated by integrating the mass flux across the exit plane of the nozzle, Eqn. 4.3.

\[ \dot{m} = \int_A \rho (U \cdot n) \, dA \] (4.3)

Specific impulse efficiency normalizes the simulated specific impulse by that predicted by quasi-1D inviscid theory. As such, specific impulse efficiency sheds light on the effect of viscous losses and how those losses degrade performance. See Eqn. 4.4.

\[ I_{sp}^{eff} = \frac{I_{sp}}{I_{sp}^{1D}} \] (4.4)

The term \( I_{sp}^{eff} \) calculated using the same formula as \( I_{sp} \), however, the thrust and mass flow rate calculated from simulation are replaced with the quasi-1D counterparts. Utilizing the isentropic relations for compressible flow and knowing the propellant properties and flow conditions, the quasi-1D thrust and mass flow rate can be estimated from Eqns. 4.5 and 4.6.

\[ F^{1D} = A_t p_0 \sqrt{\frac{2 \gamma^2}{\gamma - 1}} \left( \frac{2}{1 + \gamma} \right)^{\frac{1}{\gamma - 1}} \left[ 1 - \left( \frac{p_e}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} \right] + A_t (p_e - p_\infty) \] (4.5)
4.2. GRID ARCHITECTURE

\[ \dot{m}^{1D} = A_t p_0 \gamma \sqrt{\frac{(\frac{2}{\gamma+1})^{\frac{\gamma+1}{\gamma-1}}}{\gamma R T_0}} \]  

(4.6)

4.2 Grid Architecture

For the purposes of thrust calculation the region around the linear-walled nozzle exit plane and the plug nozzle spike are primarily of interest. To ensure an accurate solution in these areas additional portions of the nozzle and an extended down-stream region must be included in the computational domain (29). The computational domain encompasses half of each nozzle, leveraging symmetry to reduce cell count and simulation run-time. All meshes are composed of structured blocks containing hexahedral cells and are designed with mesh-flow alignment in mind. As such, to model the non-rectangular computational domain several individual structured mesh blocks must be used. The block structure of the both the linear-walled and plug nozzle configurations is presented in Fig. 4.1. The linear-walled and plug geometries are divided into 5 and 7 mesh blocks respectively. These architectures are used to maintain smooth transitions in cell size and orientation throughout the domain.

OpenFOAM® runs exclusive 3-dimensional meshes. To run a 2D simulations in OpenFOAM® a pseudo-3D meshes is generated by extruding a 2D dimensional mesh to be one cell think in the 3D dimension. A mesh solver is applied to create smooth transitions between blocks. A Multigrid solver with Thomas-Middlecoff interior control function and a Steger-Sorsen boundary control function is used. A relaxation factor of 0.9 and prolongation factor 0.6 are used. At block boundaries, orthogonal angle control and floating point distributions are imposed. Meshes are solved for
4.2. GRID ARCHITECTURE

around 30 iterations. Images of a coarse plug nozzle mesh ($\epsilon = 3.19$) before and after application of the mesh solver are shown in Fig. 4.2. Maximum equiangle skew is below 0.45 with average skew usually in the range 0.05-0.10 depending upon the mesh refinement and nozzle geometry.

![Figure 4.1: Mesh block architecture.](image1)

![Figure 4.2: Grid before and after mesh solver.](image2)
4.3 GRID GENERATION

Computational meshes are created using Pointwise® mesh generation software. Pointwise® gives the user the ability to create meshes interactive through the gui environment, however, the user also has the option to automate the meshing processing using Pointwise®’s Tcl based scripting language Glyph.

In this study, two classes of geometries are examined, linear-walled and plug, with up to 8 variation. Additionally several refinements of each variation might also be generate to assess grid sensitivity. As a result, in this study, the meshing process is automated to streamline simulation run transition and reduce the required user-interface hours per run. For linear-walled nozzles, each mesh is defined by a total of 3 user-defined input parameters: the expander angle $\alpha$, the expansion ratio $\epsilon$, and the number of points used to span the nozzle throat $N_p$. For plug nozzles those parameters are: percent length $\%L$, expansion ratio $\epsilon$, and the number of points used to span the nozzle throat $N_p$. These parameters are fed into a Python wrapper which writes a Glyph script that is then imported in to Pointwise® to generate the mesh. The Python® wrapper then creates the run directory, runs the simulation, and finally calculates and catalogs nozzle performance. This process is looped over several nozzle geometries utilizing past solution to initialize new runs.

For example, the run set for the linear-walled nozzle with $\epsilon = 3.19$ begins by running with the largest expansion angle $\alpha = 50^\circ$ for all 4 inlet pressure $p_0 = 25-200$. The $50^\circ$ solutions are then mapped onto $45^\circ$ meshes and then those simulation are run to convergence. This process is repeated, stepping down the expander half-angle by $5^\circ$ each iteration of the automation loop.
4.3. GRID GENERATION

Figure 4.3: Coarse mesh with automated connector spacing labeled.
4.4 GRID SENSITIVITY

Of the three parameters, the mesh density is controlled primarily by \( N_p \), the number of points across the throat. This is achieved by basing the cell size in other location of the mesh by the cell size at the throat. A schematic of a coarse mesh is shown in Fig. 4.3. Connectors with \( N_p \) points are shown as red lines. Imposed cell size at some sample locations are shown in blue for which, \( l^* \) is the width of the nozzle throat, \( l_{in} \) is the width of the nozzle inlet, and \( \epsilon \) the expansion ratio of the nozzle. These algebraic rules are implemented within the Python® environment during the Glyph script writing process. As \( N_p \) is increased the cell size is uniformly decreased across the mesh.

4.4 GRID SENSITIVITY

This method, of successive uniform refinement based on \( N_p \) is used to conduct grid sensitivity studies for plug and linear-walled geometries. During simulation, intermediate solutions are written at user prescribed time intervals. For the psuedo-time-stepping scheme implemented, these intermediate solutions are used to judge the convergence of the solution on a particular mesh. For each write interval the thrust and mass flow are calculated and used as metrics to assess convergence. Typically for coarser meshes the convergence criterion placed on thrust and mass flow is less stringent; however, for final refinements a temporal convergence criterion of less than 0.01% change in thrust and mass flow is typical. Thrust and percent difference in thrust are plotted in Fig. 4.4 for approximately 10 write intervals of 3 example meshes with \( N_p = 15, 45, \) and 60. Black horizontal lines indicate where a coarser solution is mapped onto the finer mesh. It can be seen for each mesh there is an initial spike/dip in thrust.
4.4. GRID SENSITIVITY

followed by convergence to a fixed value within 10 write intervals. Additionally, a less than 1% percent difference in thrust is experience across the full range of refinements.

Grid sensitivity studies for plug and linear-walled geometries with $\epsilon = 6.22$ are shown in Fig 4.5. Here only the final converged write interval is used for each mesh. Percent difference in calculated mass flow and thrust, with respect to the most refined case, are plotted versus the number of cells. For the plug geometry $N_p$ values of 20, 30, 40, 50, and 60 are shown. For the linear-walled nozzle, $N_p$ values of 20, 30, 40, and 50 are shown. For both geometries the final refinement step yields a less than 0.2% change in thrust. The final mesh for the plug configuration contained 60 cells across the throat with approximately 150,000 cells while linear-walled geometries contained 50 cells across the throat with approximately 50,000 cells.
4.4. GRID SENSITIVITY

Figure 4.4: Convergence temporally for different mesh sizes.

Figure 4.5: Grid sensitivity study results.

(a) Plug, $\epsilon = 6.22$

(b) Linear walled, $\epsilon = 6.22$
Chapter 5

Results

5.1 Linear-Walled Nozzles

In this study, linear-walled geometries are used as a performance baseline to assess the merit of plug nozzle designs for microscale applications. As such a study, examining the effects of expander half-angle $\alpha$ on linear-walled nozzle performance, is conducted for the two expansion ratios of interest ($\epsilon = 3.19$, and 6.22). Mach contours within the linear-walled nozzles are shown in Figs. 5.1 and 5.2 for expansion ratios of 3.19 and 6.22 respectively. It can be seen that nozzles with larger half-angles have a shorter expander section, and viscous losses associated with fluid-surface interactions are traded for transverse losses associated with flow misalignment at the nozzle exit. For a given Reynolds number and expansion ratio, these two competing effects can be balanced to yield an optimal expansion angle.
5.1. LINEAR-WALLED NOZZLES

Figure 5.1: Linear-walled nozzle mach contours $\epsilon = 3.19$
5.1. LINEAR-WALLED NOZZLES

(a) $\alpha = 15^\circ$, $Re = 80$

(b) $\alpha = 15^\circ$, $Re = 640$

(c) $\alpha = 30^\circ$, $Re = 80$

(d) $\alpha = 30^\circ$, $Re = 640$

(e) $\alpha = 45^\circ$, $Re = 80$

(f) $\alpha = 45^\circ$, $Re = 640$

Figure 5.2: Linear-walled nozzle mach contours $\epsilon = 6.22$
5.1. LINEAR-WALLED NOZZLES

Plots of specific impulse efficiency versus the nozzle half-angle are shown in Fig. 5.3 and can be used to identify the optimal half-angle. In general, lower Reynolds numbers show a more well defined peak in performance while higher Reynolds numbers tend to plateau for small half-angles. Additionally, the performance at different Reynolds number tends to converge for large half-angles, in which transverse losses dominate viscous losses. Transverse losses are largely a function of half-angle while viscous losses are a function of Reynolds number and expander length. By reducing the length of the expander viscous losses are negated and the performance dependence of the nozzle on Reynolds number is likewise reduced.

For an expansion ratio of 3.19 the optimal half-angle is found to be 30, 25, 25, and 20° for Reynolds number of 80, 160, 320, and 640 respectively. For an expansion ratio of 6.22 the optimal half-angle is found to be 35, 35, 30, and 25° for Reynolds number of 80, 160, 320, and 640 respectively. In general the increased length of the larger expansion ratio nozzle necessitates a larger half-angle to optimize performance. Additionally, a 30° half-angle appears to provide good performance over the full range of expansion ratios and Reynolds number. This result agrees with a number of past studies examining micronozzle half-angle (16; 32).

Specific impulse efficiency can also be used to quickly gauge magnitude of the combined effects of transverse losses and viscous losses. As a quick recap, specific impulse efficiency is the specific impulse calculated from simulation normalized by the theoretical maximum predicted by quasi-1D inviscid theory. See Eqns. 4.4, 4.5, and 4.6 for mathematical definitions. From Figs. 5.3(a) and 5.3(b), it can be seen that performance losses in the range of 4-21% are experienced due to the combination of viscous and geometric effects.
5.1. LINEAR-WALLED NOZZLES

Figure 5.3: Specific impulse efficiency of linear-walled nozzles

(a) $\epsilon = 3.19$

(b) $\epsilon = 6.22$
### 5.1. LINEAR-WALLED NOZZLES

Specific impulse efficiency, however, is a poor metric for comparing nozzles of different expansion ratios because the quasi-1D inviscid specific impulse is a function of the expansion ratio. As a result, the calculated specific impulse of an $\epsilon = 3.19$ nozzle would be normalized by a 1D inviscid specific impulse of approximate 134 sec whereas an $\epsilon = 6.22$ nozzle would be normalized by 141 sec. In this regard, a smaller expansion ratio nozzle is given an unfair advantage by this metric and thus, to compare the performance of the two expansion ratio nozzles, the specific impulse is used instead of the specific impulse efficiency.

Specific impulse for both expansion ratios is plotted in Fig. 5.4. Here particular colors correspond to particular Reynolds numbers. Circular markers are used for $\epsilon = 3.19$ nozzles and square markers are used for $\epsilon = 6.22$ nozzles. For larger Reynolds numbers, $Re = 320$ and $640$, the $\epsilon = 6.22$ nozzles outperform the $\epsilon = 3.19$. Conversely, at the lowest Reynolds number examined, $Re = 80$, the $\epsilon = 3.19$ nozzle outperforms the $\epsilon = 6.22$. This should be expected given that Reynolds number is controlled by the pressure ratio and that the design pressure ratios of the $\epsilon = 3.19$ and $\epsilon = 6.22$ nozzles are different. The $\epsilon = 3.19$ nozzle is optimally expanded at a Reynolds number slightly less than 80 (PR≈ 20) while the $\epsilon = 6.22$ nozzle delivers optimal expansion at a Reynolds number of 160 (PR≈ 50). Thus, at a Reynolds number of 80 the $\epsilon = 3.19$ nozzle delivers near optimal expansion of the exhaust gases while the larger $\epsilon = 6.22$ nozzle over-expands the gas delivering poorer performance. Similarly at high Reynolds number the larger expansion ratio nozzle is less under-expanded and can take better advantage of the large pressure ratio. The interesting performance exchange between the two nozzles occurs at a Reynolds number of 160 (PR≈ 50). Here the $\epsilon = 6.22$ nozzle is at its design condition and therefore
5.1. LINEAR-WALLED NOZZLES

should exhibit near optimal expansion while the $\epsilon = 3.19$ nozzle is under-expanded and thus does not take full advantage of the large pressure ratio. As a result from quasi-1D inviscid theory one would expect the $\epsilon = 6.22$ nozzle to outperform the smaller $\epsilon = 3.19$ nozzle; however, their performances are fairly similar exhibiting a cross over at a half-angle of 30°. In this case, for small half-angles, the additional incurred viscous losses of the longer $\epsilon = 6.22$ nozzle negates the benefits of a larger design pressure ratio.

![Graph showing specific impulse efficiency of linear-walled nozzles](image_url)

*Figure 5.4: Specific impulse efficiency of linear-walled nozzles*
5.2 Plug Nozzles

One the microscale, plug nozzle performance shows many of the same characteristics as that of linear-walled nozzles. The theme of balancing viscous and geometric losses is still dominant and as such macroscale designs are modified to suite the microscale flow regime. For linear-walled nozzles this involves an increase in half-angle between macro and microscale designs. In this study, a full length Angelino contour is adapted via plug-length reduction as described in Section 2.3. Analogous to increasing linear-walled half-angle, plug-length reduction is implemented with the goal of mitigating viscous losses. This is accomplished through three different methods: scaling, parabolic, and truncation. In all three cases, however, length reduction leads to increased transverse losses again analogous to the increased half-angle linear-walled designs. Plots of plug arc length and tip angle, for parabolic and scaled plugs, are shown in Fig. 2.7. Nozzles with shorter arc length have less surface area decreasing viscous losses, however, the larger tip angle orients the flow towards the nozzles center-line increasing transverse losses.

Performance of Shortened Nozzles

Using specific impulse efficiency as a metric, performance is examined for scaled (blue), parabolic (black), and truncated plugs (orange), in Figs. 5.5(a) and 5.5(b) for effective expansion ratio of $\epsilon = 3.19$ and $\epsilon = 6.22$ respectively. For each plot, results for different Reynolds numbers are indicated by different marker shapes. Note the different ranges for of the x-axes between the two plots. In general, low Reynolds number favor shorter nozzle designs and high Reynolds numbers favor long nozzle
5.2. **PLUG NOZZLES**

designs that are more closely related to the full length Angelino contour. Additionally, for a given Reynolds number, the performance of scaled and parabolic nozzles tends to peak at a longer percent length than truncated designs. For all Reynolds number and both expansion ratios, the best performance is achieved by the scaling method. For a Reynolds number of 640 this occurs at 50% length for both expansion ratios. At a Reynolds number of 80 it occurs at 30% for the $\epsilon = 3.19$ nozzle and 20% for the $\epsilon = 6.22$ nozzle. Over the full range of Reynolds numbers and expansion ratios, a 40% scaled nozzle provides a good balance of performances, with a 20% truncated nozzle providing comparable though slightly worse performance.

For all Reynolds number and expansion ratios examined, plug nozzle performance is increased by shortening the Angelino contour. The benefit derived from shortening the plug is quantified by the percent increase in specific impulse relative to the full length plug nozzle. The percent increase in performance is shown in Figs. 5.6(a) and 5.6(b) for $\epsilon = 3.19$ and 6.22 respectively. Almost all shortened geometries outperform the full length geometry from which they are derived. The exceptions are the shortest plugs operating at the highest Reynolds number. At high Reynolds number the benefit is modest, in the range of 1-2% increase in performance at Re = 640; however, as Reynolds number is reduced the benefit grows non-linearly. At a Reynolds number of 80 the performance can be improved by 10-14% using the scaling method. The $\epsilon = 6.22$ nozzle typically can benefit more from length reduction at a given Reynolds number. This is because the full length nozzle at $\epsilon = 6.22$ is longer than the full length nozzle at $\epsilon = 3.19$ as to achieve the desired amount of flow turning for the effective expansion ratio. As a result, viscous losses are greater for the full length $\epsilon = 6.22$ and there is more room for performance increase via shortening.
5.2. **PLUG NOZZLES**

Isolines of Mach number equal to 1 are shown in Fig. 5.7 to demonstrate the reduction in subsonic boundary layer caused by plug shortening. For the 40% length nozzle (bottom) the subsonic boundary layer growth with increasing Reynolds number is largely subdued in comparison to the full length nozzle (top). It is this containment of boundary layer that makes plug shortening beneficial on the microscale. Examining the Mach contours of Figure 5.8 it can be seen that the larger boundary layer for long nozzles tends to push the supersonic flow away from the plug causing the plume to bulge downstream. For the 40% length nozzle at a Reynolds number of 80 (Fig 5.8(c)) this boundary layer is reduced yielding better flow alignment and a plume nearly free of compression-expansion structure. At a Reynolds number of 640 (right) the boundary layer is much smaller and plays a less significant role. As such the shorter 40% and 30% length nozzles cause the two high-speed jets to impinge and shocks to develop caused by the large tip angle of the plug contours. The transverse losses caused by the jet impingement angle is what limits the beneficial amount of plug-length reduction.
5.2. **PLUG NOZZLES**

Figure 5.5: Specific impulse efficiency of plug nozzles

(a) $\epsilon = 3.19$

(b) $\epsilon = 6.22$

*Figure 5.5: Specific impulse efficiency of plug nozzles*
5.2. PLUG NOZZLES

Figure 5.6: Increase in specific impulse due to plug length reduction

(a) $\epsilon = 3.19$

(b) $\epsilon = 6.22$
5.2. PLUG NOZZLES

Figure 5.7: Contours of Mach equals 1 at different Reynolds numbers, $\epsilon = 3.19$
5.2. PLUG NOZZLES

Figure 5.8: Scaled plug nozzle mach contours $\epsilon = 3.19$
5.2. PLUG NOZZLES

Comparison: Scaled and Parabolic Nozzles

The plug contours generated by the scaling and parabolic methods are fairly similar, however the performance discrepancy between the two methods is non-trivial. Parabolic nozzle under-perform scaled nozzles. The level of under-performance is larger for long nozzles and converges at a nozzle length of 30%, at which point the parabolic and scaling methods produce identical plug contours.

![Sample Location](a)

![Pressure](b)

![Velocity Components](c)

![Temperature](d)

*Figure 5.9: Comparison of scaled and parabolic plug nozzle exit states for 60% length, $\epsilon = 3.19$, and $Re = 80$*

To examine the difference, field data is sampled along the “exit plane” of the 60%
5.2. PLUG NOZZLES

![Diagram showing nozzle contour and magnitude of curvature.](image)

Figure 5.10: Curvature of parabolic and scaled contours

length scaled and parabolic nozzles at an expansion ratio of 3.19. The 60% length nozzles were selected because they differ the most in geometry and performance, and therefore any flow-field differences should be the most exaggerated and easiest to spot. The location of the sampling line is shown in red in Fig. 5.9(a). Plots of pressure, velocity, and temperature are shown in Figs. 5.9(b), 5.9(c), and 5.9(d). In the plots $x = 0$ corresponds to the nozzle centerline, i.e. where the sample line meets the nozzle tip. It can be seen the parabolic nozzle produces a lower velocity and a higher temperature at the nozzle exit. This suggests that parabolic nozzles are less efficient in converting the thermal energy of the chamber into kinetic energy at the exit, thus reducing performance in comparison to scaled nozzles.

To examine the cause of the performance disparity, the curvature profiles of parabolic and scaled nozzles are compared. Curvature as a function of the $y$-distance...
5.2. **PLUG NOZZLES**

from the nozzle symmetry plane is plotted in Fig. 5.10(b) for the 60% scaled, 60% parabolic, and full length nozzle contours. It can be seen the parabolic design (blue) has a much larger curvature at the throat. As a result, parabolic contours turn the supersonic core of the flow too rapidly in the throat region causing a compression wave to form. For the 60% length parabolic nozzle this compression wave can be seen in the pressure profile of Fig. 5.9(b), in which there is a distinct bump for the parabolic nozzle. Conversely, scaled nozzles show a similar curvature profile to the full length Angelino nozzle. Length reduction through scaling reduces the near-throat curvature of the plug profile. This offsets the contour inwards compensating for the boundary layer growth at low Reynolds numbers and allowing the supersonic core to follow a more isentropic flow path.

**Truncated Nozzles**

Plug truncation seeks to increase performance by removing the long straight portion of the of the full length Angelino curve near the tip. This portion of the plug wall has surface normals nearly perpendicular to the the flow direction and as a result little thrust is delivered and large viscous losses are incurred. Plug truncation, however, yields a large flat base who’s influence on the flow field can degrade performance. As shown in 5.5(a), truncated nozzles improve performance for the Reynolds number Regime examined, however, their performance is typically inferior to that of scaled nozzles. Mach contours for truncation lengths ranging from 8-60% are shown in Fig. 5.11. Strong expansion-compression structures can be seen in the plume of the shorter nozzles as the two jets turn around the base and impinge.

The flow-field upstream of the truncation however remains largely the same as that
5.2. **PLUG NOZZLES**

of the full length Angelino nozzle. Isolines of Mach number equal to one are plotted in Fig. 5.12 for scaled and truncated nozzles. For the Truncated nozzles it can be seen that the $M = 1$ contours coincide with that of the full length nozzle up until the point of truncation. This indicates that the boundary layer growth is identical and that the supersonic core is still being pushed up and away from the surface of truncated plugs. The scaled geometries however, compensate for the boundary layer growth and the $M = 1$ isoline can be seen to gradual move inwards with increasing length reduction. Full Mach contours for the $\epsilon = 6.22$ plug nozzles at $Re = 80$ are shown for corresponding truncation and scaled lengths in Fig. 5.13.
5.2. **PLUG NOZZLES**

![Images of truncated plug nozzle mach contours](image)

(a) 60% length  
(b) 50% length  
(c) 40% length  
(d) 30% length  
(e) 20% length  
(f) 00% length

*Figure 5.11: Truncated plug nozzle mach contours $\epsilon = 3.19$ and $Re = 80$*
5.2. PLUG NOZZLES

Figure 5.12: Contours of Mach equals 1 for different length nozzles at \( \epsilon = 3.19 \) and \( Re = 80 \)
5.2. **PLUG NOZZLES**

![Plugs nozzle mach contours](image)

(a) truncated 60% length  
(b) scaled 60% length  
(c) truncated 40% length  
(d) scaled 40% length  
(e) truncated 20% length  
(f) scaled 20% length

*Figure 5.13: Plug nozzle mach contours $\epsilon = 6.22$ and $Re = 80$*
5.3 **Plug and Linear-Walled Comparison**

In Fig. 5.14 the specific impulse of linear-walled and plug nozzles designs are compared. For each class of nozzle (i.e. linear-walled, scaled plug, truncated plug, and parabolic plug) the design which provides the best balance of performance over the Reynolds number range $Re = 80 - 640$ is selected for comparison. For linear-walled designs this is an expander half angle of 30°, for scaled/parabolic plugs percent lengths of 40% and 30% for $\epsilon = 3.19$ and 6.22 respectively, for truncated plugs a percent length of 20%. In Fig. 5.14(a) performance is compared for nozzle with an expansion ratio of 3.19. For these nozzles, plug designs provide better performance for the full Reynolds number range, with scaled and parabolic nozzles outperforming their truncated counterparts. The difference in performance between the linear-walled design and the plug nozzles seems to converge at the lowest Reynolds number $Re = 80$. In Fig. 5.14(a) performance is compared for nozzle with an expansion ratio of 6.22. For this expansion ratio the plug nozzle yield better performance at Reynolds number of 160, 320 and 640 while the linear-walled design yields better performance at a Reynolds number of 80. Additionally at high Reynolds numbers the truncated design outperforms the parabolic and scaled designs.

It should be noted that the only case in which the linear-walled design outperforms the plug nozzle designs is during a state of over-expansion. The $\epsilon = 6.22$ nozzles experience a transition from over-expanded to under-expanded at a Reynolds number of about 160 (PR≈50). This transition appears to coincide with the performance cross-over between linear-walled and plug nozzles designs. Moreover, for the Reynolds number range $Re = 80 - 640$ the $\epsilon = 3.19$ nozzles are always under-expanded, how-
5.3. **PLUG AND LINEAR-WALLED COMPARISON**

(a) $\epsilon = 3.19$

(b) $\epsilon = 6.22$

*Figure 5.14: Specific impulse comparison between linear-walled and plug designs*
5.3. **PLUG AND LINEAR-WALLED COMPARISON**

![Figure 5.15: Mach=1 isolines for a 40% length $\epsilon = 6.22$ plug nozzle.](image)

ever, they transition to over-expansion at a Reynolds number just below $Re = 80$. In Fig. 5.14(a) the performances appear to be converging in this region suggesting a similar performance cross-over might occur for $\epsilon = 3.19$ nozzles at the transition between under and over-expansion states. The poor plug nozzle performance appears during over-expansion appears to be caused by a rapid growth of the subsonic boundary layer. Mach 1 isolines are plotted for a 40% length $\epsilon = 6.22$ plug nozzle in Fig. 5.15. For Reynolds number of 160-640 (all in the under-expanded state) the extension of the subsonic boundary layer past the nozzle tip does not change much. At a Reynolds number of 80 however the subsonic boundary layer grows significantly extending further downstream. In comparison for the 40% length, $\epsilon = 3.19$ plug nozzle there is a negligible growth in subsonic boundary layer for all Reynolds number 80-640. See Fig. 5.7(b).
5.4. SLIP EFFECTS

To examine the validity of the continuum model and boundary conditions selected, the Knudsen number throughout the flow is examined. Contours of Knudsen number for linear-walled and plug geometries for Reynolds numbers of 80 and 640 are shown in Figure 5.16. Note the logarithmic scale.

At a Reynolds number of 80, the Knudsen number in the chamber (upstream of the throat) is approximately 0.02. For the plug nozzle at $Re = 80$, a sizable
5.4. SLIP EFFECTS

region near the tip exceeds a Reynolds number of 0.2. Additionally the linear-walled geometry experiences Knudsen numbers in excess of 0.5 near the nozzle lip at \( Re = 80 \). These Knudsen numbers are in excess of the continuum thresholds proposed by Xie (average Kn<0.01), and Liu et al. (local Kn<0.045) (23; 24). This suggests that the no-slip boundary condition does not accurately model the flow physics at these scales. The implementation of slip however, may still provide an accurate prediction of performance (25).

At a Reynolds number of 800, the Knudsen number in the chamber is below 0.001. In the supersonic region of both the linear-walled and plug nozzle the Knudsen number is primarily in the region 0.01-0.1 which indicates that slip-effects along the wall could influence nozzle performance.

A Maxwell slip boundary condition is thus implemented on the nozzle walls to examine the impact on performance of the different boundary conditions. A tangential momentum accommodation coefficient (TMAC) of 0.85 is selected consistent with polished silicon. The specific impulse of the full length and 40\% length scaled \( \epsilon = 6.22 \) plug nozzles is compared in Fig. 5.17 for no-slip and 1\(^{st}\) order Maxwell slip boundary conditions. It can be seen the same qualitative trends exist for the slip and no-slip cases; length reduction yields a performance increase. However, the benefit of the length reduction appears to be over-estimated by the no-slip boundary condition. The percent increase in performance caused by shortening the full length nozzle to 40\% by the scaling method is shown in Fig. 5.18 for both slip and no-slip cases. It can be seen the slip boundary condition predicts a more modest benefit due to shortening, in the range of 2-8\%, while the no-slip condition predicts a performance increase in the range of 2-12\%. This is caused by full length nozzle’s larger surface
5.4. SLIP EFFECTS

Figure 5.17: Wall slip effects on specific impulse for plug $\epsilon = 6.22$
5.4. SLIP EFFECTS

Figure 5.18: The percent increase in specific impulse caused by shortening the full length $\epsilon = 6.22$ plug to 40\% the original length via the scaling method is plotted for slip and no-slip boundary conditions.

area which accrues more viscous losses than the shorter 40\% length nozzle. When a slip boundary condition is imposed, the longer full length geometry is affect more than the shorter 40\% geometry and the performance gap between the two begins to close.

The percent difference between the slip an no-slip specific impulse is plotted in Fig. 5.19. For the full length geometry at the lowest Reynolds number of 80 the no-slip simulation under-predicts specific impulse by about 5\%, however for all other cases shown slip and no-slip simulations agree within 2\%.
Figure 5.19: Percent difference between slip and no-slip specific impulse for the full length and 40% scaled $\epsilon = 6.22$ plug nozzles.
Chapter 6

Conclusion

The nozzles used in micropropulsion systems encounter a fundamentally different flow regime than their macro-scale counterparts. As such, high performance micronozzle designs can differ significantly from the well-understood classic designs for macroscale nozzles. In this study, the macro-scale plug nozzle design method proposed by Angelino is adapted to yield higher performance on the microscale via three plug shortening techniques: 1) a parabolic contour, 2) truncation, 3) a geometric transformation involving scaling. The performance of nozzles designed using these methods was then compared for different shortened lengths ($\%L = 10 – 60\%$) and range of Reynolds numbers ($Re = 80 – 640$) using 2D computational fluid dynamic simulation. The highest performing length for each of three shortening techniques was then selected for comparison to a linear-walled nozzle with equivalent expansion ratio and throat dimension.

For the Reynolds number range examined, the full length Angelino plug design provides poor performance, incurring excessive viscous losses due to its long length. Shortening boosts performance by reducing the surface area of the nozzle plug thus re-
ducing viscous losses. Shortened geometries, however, orient the flow inward towards the nozzle centerline causing transverse losses due to flow misalignment. As such an optimum amount of shortening can be found which balance viscous and transverse losses.

All three shortening methods provide an increase in performance, the magnitude of which is highly dependent upon the Reynolds number. At high Reynolds number \((Re = 640)\) the increase in performance due to shortening is modest in the range of \(1-2\%\); however, for the lowest Reynolds number examined \((Re = 80)\) no-slip simulations predict that the benefit can be as large as \(14\%\), depending upon the shortening method and the amount of length reduction. These figures are slightly reduced when a wall slip is incorporated in the model, with a maximum benefit due to shortening near \(8\%\); however qualitatively the results were not affected when switching from no-slip to slip. Results suggest that, of the three shortening method, the geometric scaling technique generally produces the highest performance. In reality however, nozzle design is not driven purely by performance, and structural integrity as well as heat transfer concerns might warrant the small drop in performance when switching from a scaled to truncated designs. Parabolic nozzles however, are found to be universally inferior to scaled nozzles and if the manufacturing technique is capable of delivering the precision necessary to distinguish the two contours a scaled approach would be preferable.

In general the optimal amount of shortening was dependent upon the Reynolds number and the shortening technique. Scaled and parabolic nozzles generally achieve a maximum in performance for a shorter percent length than truncated designs. For the 3.19 expansion ratio nozzles, truncated nozzles at 20\% and scaled/parabolic noz-
zles 40% length deliver good performance for full range of Reynolds numbers. For an expansion ratio of 6.22 lengths of 20% and 30% yield good overall performance for truncated and scaled/parabolic designs respectively. It should be noted however, for the $\epsilon = 6.22$ plug nozzle the 20% truncated case was the shortest case examined and further truncation might have a positive impact on performance. The 6.22 expansion ratio nozzles favors shorter nozzles than their 3.19 expansion ratio counterparts because the longer 6.22 nozzle was more heavily influenced by viscous losses.

Plug designs were found to outperform linear-walled designs when in a state of under-expansion, however, for the over-expanded case examined, the linear-walled geometry outperformed all three plug nozzle types. This is interesting because plug nozzles were originally developed on the macroscale to provide high performance when in a state of over-expansion avoiding issues associated with flow separation for SSTO launch systems. On the microscale however, over-expanded plugs appear to have a dramatic increase in the size of the subsonic boundary layer which extended a significantly further length down-stream. As such, the results suggest that plug nozzle incur substantial viscous penalties when over-expanded; however, only one over-expanded case was examined in this study. The over-expanded regime would need to be probed more thoroughly to demonstrate this performance trend definitively.

It should be noted however, that these micronozzle systems are being developed for attitude control systems of space vehicles. Thus, under-expansion is not really representative of the flow state of an actual nozzle which would see a vacuum back-pressure. As a result, the plug nozzles’ superior performance when under-expanded suggests that they are a good design option for micropropulsion applications.


selection for precision formation flying satellites. AIAA-2001-3646.


BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


